

Solving Quadratic Equations without Factoring

Recall: To solve by factoring,

- (1) collect all terms on one side of equal sign
- (2) factor the expression
- (3) use $(a)(b) = 0$ to state $a = 0$ or $b = 0$

Consider this example:

$$x^2 - 12x + 32 = 0$$

$$(x - 8)(x - 4) = 0$$

$$x - 8 = 0 \quad \text{or} \quad x - 4 = 0$$

$$x = 8 \qquad \qquad \qquad x = 4$$

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Vertex form can also be very useful for solving a quadratic equation.

- Ex.1 (a) Write $y = x^2 - 12x + 32$ in vertex form
 (b) Solve for $y = 0$

$$\begin{aligned} \text{(a)} \quad & y = x^2 - 12x + 32 & -\frac{12}{2} = -6 \\ & y = x^2 - 12x + 36 - 36 + 32 & (-6)^2 = 36 \\ & y = (x-6)^2 - 4 \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad & \text{set } y = 0 \\ & 0 = (x-6)^2 - 4 \end{aligned}$$

$$\begin{aligned} & (x-6)^2 - 4 = 0 \\ & +4 \quad +4 \\ & (x-6)^2 = 4 \\ & \sqrt{(x-6)^2} = \pm\sqrt{4} \\ & x-6 = \pm 2 \\ & x-6 = 2 \quad \text{or} \quad x-6 = -2 \\ & \boxed{x=8} \quad \boxed{x=4} \end{aligned}$$

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In some cases, one may be simpler than the other.

Ex.2 Write in factored & vertex form, then choose which to use for solving.

$$(a) x^2 + 3x - 4 = 0 \quad \left. \begin{array}{l} S \ 3 \\ P \ -4 \\ I \ 4, -1 \end{array} \right\}$$

$$x^2 + 3x - 4 = 0$$

$$\underline{x^2 + 3x + 2.25 - 2.25 - 4 = 0}$$

$$(x+1.5)^2 - 6.25 = 0$$

$$(x+1.5)^2 = 6.25$$

$$x+1.5 = \pm\sqrt{6.25}$$

$$x+1.5 = \pm 2.5$$

$$\left. \begin{array}{ll} x+1.5=2.5 & x+1.5=-2.5 \\ x=1 & x=-4 \end{array} \right\}$$

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Ex.2 Write in factored & vertex form, then choose which to use for solving.

$$(b) x^2 - 9 = 7$$

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If factoring is not possible use the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

which is derived from completing the square (p.337-338).

Note: To use the quadratic formula, the equation must be in standard form, $ax^2 + bx + c = 0$.

The ' \pm ' symbol means there are two solutions.

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{or} \quad x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

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Ex.3 Solve using the quadratic formula.

a) $x^2 - 4x - 3 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$a = 1 \quad b = -4 \quad c = -3$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-3)}}{2(1)}$$

$$x = \frac{4 \pm \sqrt{16 + 12}}{2}$$

$$x = \frac{4 \pm \sqrt{28}}{2}$$

$$x_1 = \frac{4 + \sqrt{28}}{2} \quad x_2 = \frac{4 - \sqrt{28}}{2}$$

$$x_1 \doteq 4.65 \quad x_2 \doteq -0.65$$

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$$\text{b) } x^2 - 2x - 5 = 0$$

$$a=1 \quad b=-2 \quad c=-5$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-5)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{4 + 20}}{2}$$

$$x = \frac{2 \pm \sqrt{24}}{2}$$

$$x_1 = \frac{2 + \sqrt{24}}{2} \quad x_2 = \frac{2 - \sqrt{24}}{2}$$

$$x_1 \doteq 3.45$$

$$x_2 \doteq -1.45$$

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Assigned Work:

p.343 # 1ad, 3, 4bdf, 5ace, 8bdf, 9ad
10cd, 14, 19*

$\sqrt{-9}$ does not exist

9(d) Solve $(4x-1)(4x-1)$

$$3x(x+4) = (4x-1)^2$$

$$3x^2 + 12x = 16x^2 - 8x + 1$$

$$0 = 13x^2 - 20x + 1$$

① factor ② CTS ③ QF

~~$\begin{matrix} \cancel{-20} \\ P \times 13 \\ \cancel{2} \end{matrix}$~~ $13(x^2 - \frac{20}{13}x)$ ugly!

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad a = 13, b = -20, c = 1$$

$$x = \frac{-(20) \pm \sqrt{(-20)^2 - 4(13)(1)}}{2(13)}$$

$$x = \frac{20 \pm \sqrt{400 - 52}}{26}$$

$$x = \frac{20 \pm \sqrt{348}}{26}$$

$$x_1 = \frac{20 + \sqrt{348}}{26} \quad x_2 = \frac{20 - \sqrt{348}}{26}$$

$x =$

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14.

$$h = -4.9t^2 + 1.2t + 15$$

(a) Set $h = 15$

$$15 = -4.9t^2 + 1.2t + 15$$

$$0 = -4.9t^2 + 1.2t$$

$$0 = t(-4.9t + 1.2)$$

$$t=0 \quad -4.9t + 1.2 = 0$$

$$-4.9t = -1.2$$

$$t = \frac{-1.2}{-4.9}$$

$$t = 0.2$$

\therefore diver was above 15m for 0.2 seconds

(b) when does diver hit water?

Set $h = 0$

$$0 = -4.9t^2 + 1.2t + 15$$

$$a = -4.9, b = 1.2, c = 15$$

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19. (a) $x = -3$ and $x = 5$

$$0 = a(x - s)(x - t)$$

$$0 = a(x - (-3))(x - 5)$$

$$0 = a(x + 3)(x - 5)$$

pick $a = 1$ (easy)

$$0 = (x + 3)(x - 5)$$

$$\boxed{0 = x^2 - 2x - 15}$$

(b) $x = \frac{2 \pm \sqrt{5}}{3}$

$$x = \frac{2 + \sqrt{5}}{3} \quad x = \frac{2 - \sqrt{5}}{3}$$

$$\begin{aligned} -b &= 2 & 2a &= 3 & b^2 - 4ac &= 5 \\ b &= -2 & a &= 1.5 & (-2)^2 - 4(1.5)c &= 5 \\ & & & & 4 - 6c &= 5 \\ & & & & -6c &= 1 \\ & & & & c &= -\frac{1}{6} \end{aligned}$$

$$1.5x^2 - 2x - \frac{1}{6} = 0 \quad c = -\frac{1}{6}$$

$$9x^2 - 12x - 1 = 0$$

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$$ax^2 + bx + c = 0$$

$$a\left(x^2 + \frac{b}{a}x\right) + c = 0 \quad \left(\frac{b}{2a}\right)^2$$

$$a\left[x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} - \frac{b^2}{4a^2}\right] + c = 0 = \frac{b^2}{4a^2}$$

$$a\left[\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2}\right] + c = 0$$

$$a\left(x + \frac{b}{2a}\right)^2 - \frac{ab^2}{4a^2} + c = 0$$

$$a\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a} - c$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$$

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