Predicting the Number of Roots of Quadratics

Nov 29/2011

Recall: For a quadratic relation
roots = zeroes = x-intercept = solutions

Given vertex form, look at:
- the location of the vertex (above/below x-axis?)
- the direction of opening (up/down?)

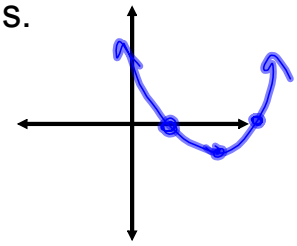
Ex.1 $y = 3(x - 5)^2 - 1$

V(5 , -1)

The vertex lies **above/below?** the x - axis.

The parabola opens **up/down?**

zero(es): 2



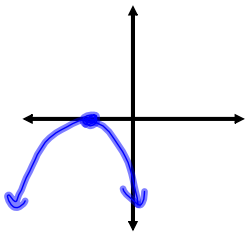
Apr 15-1:05 PM

Ex.2 $y = -2(x + 1)^2 + 0$

V(-1 , 0)

The vertex lies on the x-axis

zero(es): 1



Apr 19-8:11 PM

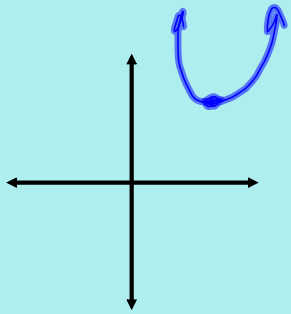
Ex.3 $y = 3(x - 5)^2 + 6$

V(5 , 6).

above/below? the x-axis

opens up/down?

zero(es): 0



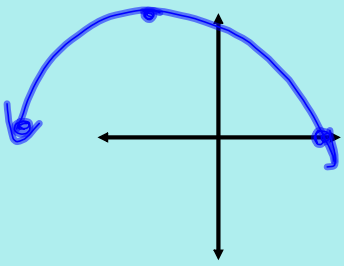
Ex.4 $y = -0.5(x + 3)^2 + 11$

V(-3 , 11).

above/below? the x-axis

opens up/down?

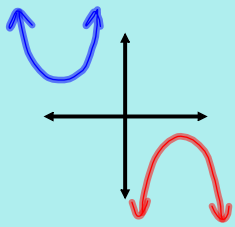
zero(es): _____



Apr 15-1:14 PM

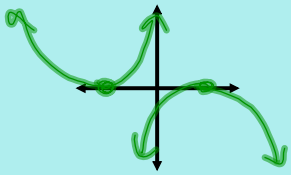
In general to identify the zeros from vertex form:

There will be 0 zeroes if the vertex is above the x-axis and the parabola opens up

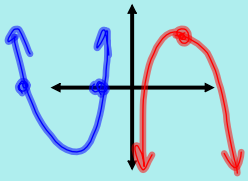


OR if the vertex is below the x-axis and the parabola opens down

There will be 1 zero if the vertex is ON the x-axis



There will be 2 zeroes if the vertex is below the x-axis and the parabola opens up



OR if the vertex is above the x-axis and the parabola opens down

Apr 15-1:16 PM

What if the quadratic is in standard form?

1. Factor and find the number of roots directly.

2. Complete the square (vertex form) and deduce the number of roots by visualizing the graph.

3. Use the quadratic formula if:
   - it cannot be factored
   - the numbers are too difficult to work with

Ex.1 Use the quadratic formula to determine the zeroes.

(a) $-2x^2 - 4x - 2 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = -2$
$b = -4$
$c = -2$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(-2)(-2)}}{2(-2)}$$

$$= \frac{4 \pm \sqrt{16 - 16}}{-4}$$

$$= \frac{4 \pm \sqrt{0}}{-4}$$

$$= \frac{4}{-4}$$

$$= -1$$

root(s): 1

root(s): $x = -1$

Apr 21-12:07 PM

(b) $y = 3x^2 - 30x + 74$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = 3$

$b = -30$

$c = 74$

$$x = \frac{-(-30) \pm \sqrt{(-30)^2 - 4(3)(74)}}{2(3)}$$

$$= \frac{30 \pm \sqrt{900 - 888}}{6}$$

$$= \frac{30 \pm \sqrt{12}}{6}$$

root(s): 2

root(s):

$$x_1 = \frac{30 + \sqrt{12}}{6} \quad x_2 = \frac{30 - \sqrt{12}}{6}$$

Apr 15-1:28 PM

(c) $y = 3x^2 - 30x + 81$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = 3$

$b = -30$

$c = 81$

$$x = \frac{-(-30) \pm \sqrt{(-30)^2 - 4(3)(81)}}{2(3)}$$

$$= \frac{30 \pm \sqrt{900 - 972}}{6}$$

$$= \frac{30 \pm \sqrt{-72}}{6}$$

root(s): 0

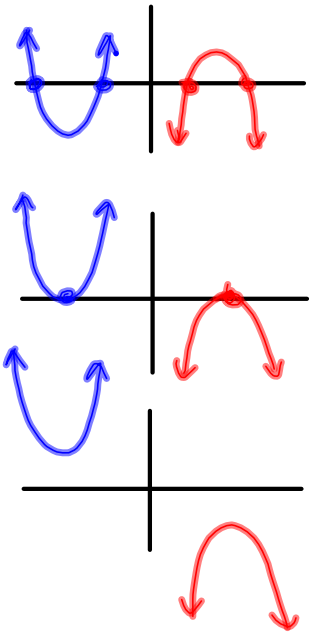
root(s): no solutions $\sqrt{-72}$ does not exist

Apr 15-1:28 PM

$D = b^2 - 4ac$ is called the discriminant.

It tells you how many zeros the quadratic has.

D is positive $b^2 - 4ac > 0$	two real roots
D is zero $b^2 - 4ac = 0$	one real root (double root)
D is negative $b^2 - 4ac < 0$	no real roots



Nov 24-1:34 PM

Ex.2 Find how many zeros each of the following quadratic relations has using the discriminant.

$D = b^2 - 4ac$

(a) $y = x^2 - 6x + 7$

$a = 1$
$b = -6$
$c = 7$

$D = (-6)^2 - 4(1)(7)$
$= 36 - 28$
$= 8$ $\therefore$ 2 roots

(b) $y = 2x^2 - 5x + 9$

$a = 2$
$b = -5$
$c = 9$

$D = (-5)^2 - 4(2)(9)$
$= 25 - 72$
< 0 $\therefore$ no roots

(c) $y = x^2 + 3x - 11$

$a = 1$
$b = 3$
$c = -11$

$D = (3)^2 - 4(1)(-11)$
$= 9 + 44$
> 0 $\therefore$ 2 roots

(d) $y = 9x^2 - 24x + 16$

$a = 9$
$b = -24$
$c = 16$

$D = (-24)^2 - 4(9)(16)$
$= 576 - 576$
$= 0$
$\therefore$ 1 root

Nov 24-1:38 PM

Assigned Work:

p.350 # 2, 3, 4, 5, 7, 9, 10, 12

Apr 19-8:15 PM