Solving Problems Using Quadratic Relations

What we have learned that we will be using:

- factoring and the quadratic formula leads to the roots

- finding the vertex (by factoring, partial factoring, or completing the square) gives you the optimal value

Remember that in word problems it is always important to identify the variables and sketching the parabola can be useful.

Apr 25-2:44 PM

Solving Problems Using Quadratic Relations

Nov30 2011

Ex.1 A hose is placed on an aerial ladder. The hose sprays water on a forest fire. The height of the water, *h*, in metres can be modelled by the relation

$$h = -2.25(d - 1)^2 + 9,$$

where *d* is the horizontal distance, in metres, of the water from the nozzle of the hose.

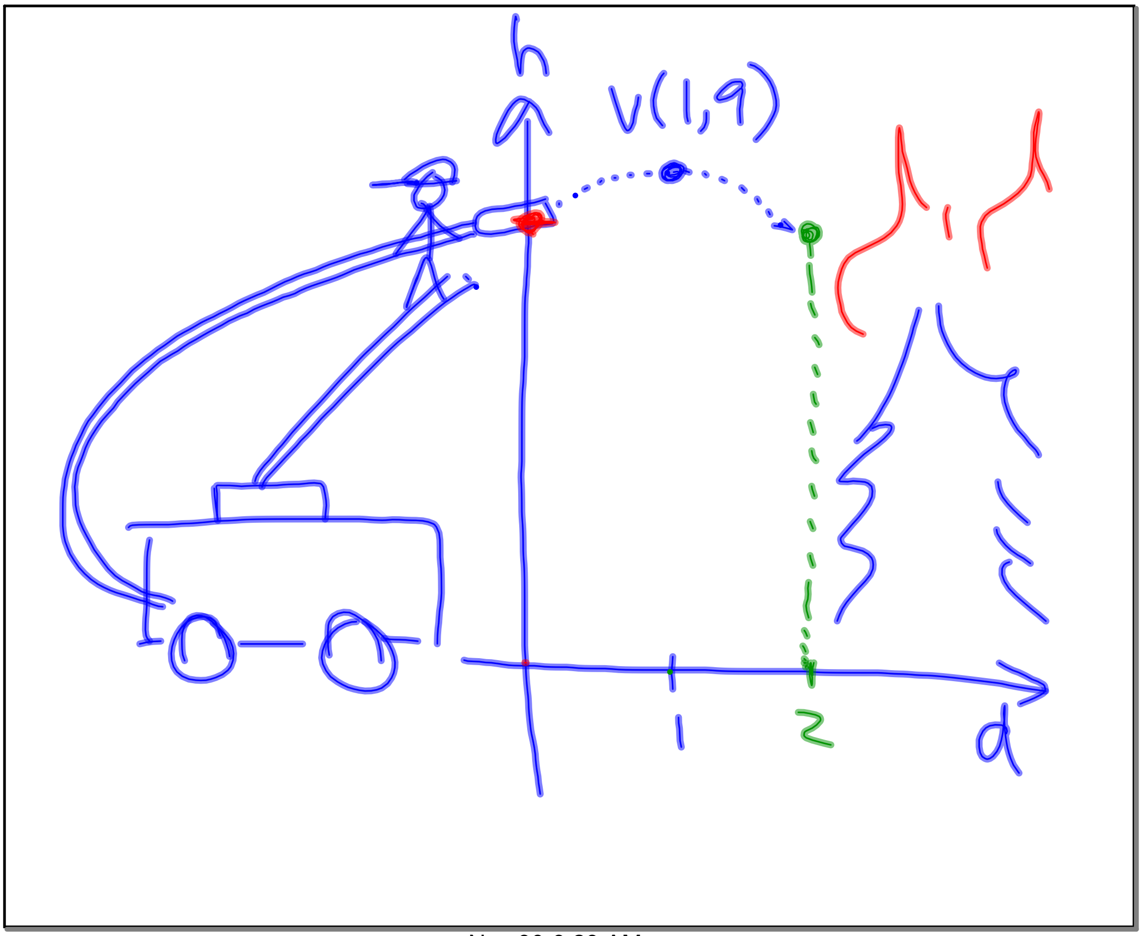
a) What is the maximum height reached by the water?

$V(1, 9)$ $\therefore$ max height is 9m

b) At what horizontal distance from the nozzle is the maximum height reached?

$V(1, 9)$ $\therefore$ max height is reached 1m from hose

Apr 25-2:56 PM



Nov 30-9:23 AM

Ex.1 A hose is placed on an aerial ladder. The hose sprays water on a forest fire. The height of the water, *h*, in metres can be modelled by the relation

$$h = -2.25(d - 1)^2 + 9,$$

where *d* is the horizontal distance, in metres, of the water from the nozzle of the hose.

c) What is the height of the aerial ladder?

set $d=0$ $\quad h = -2.25(0-1)^2+9$

$= -2.25(1)+9$

$= 6.75$

$\therefore$ height of ladder is 6.75m

d) How high is the water when it is at a horizontal distance of 2m from the nozzle?

① recognize $d=2$ is a matching point for $d=0$

$\therefore$ the water is 6.75m high when it is 2m from nozzle

② Set $d=2$

$h = -2.25(2-1)^2+9$

$= 6.75$

Apr 25-2:56 PM

Ex.2 A ball is thrown into the air. Its height, in metres, after t seconds, is $h = -4.9t^2 + 39.2t + 1.75$.

a) When does it reach maximum height?

vertex → vertex form
→ CTS.

$$h = -4.9(t^2 - 8t) + 1.75$$

$$h = -4.9[t^2 - 8t + 16 - 16] + 1.75$$

$$h = -4.9[(t-4)^2 - 16] + 1.75$$

$$h = -4.9(t-4)^2 + 78.4 + 1.75$$

$$h = -4.9(t-4)^2 + 80.15$$

$V(4, 80.15)$ ∴ max height reached after 4 seconds

b) What is the maximum height?

∴ max height is 80.15 metres.

Apr 22-9:25 PM

Ex.2 A ball is thrown into the air. Its height, in metres, after t seconds, is $h = -4.9t^2 + 39.2t + 1.75$.

c) From what height is the ball released?

y-int → $t = 0$

∴ ball is released from 1.75m above the ground

d) When does the ball hit the ground?

t $h=0$

set $h = 0$, solve for t

① $0 = -4.9t^2 + 39.2t + 1.75$
→ cannot factor
→ use quadratic formula.

② $0 = -4.9(t-4)^2 + 80.15$

$$\frac{4.9(t-4)^2}{4.9} = \frac{80.15}{4.9}$$

$$(t-4)^2 \doteq 16.357$$

$$t - 4 \doteq \pm\sqrt{16.357}$$

$$t - 4 \doteq \pm 4.044$$

$t - 4 \doteq 4.044$ or $t - 4 \doteq -4.044$

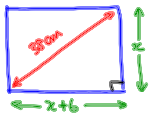
$t \doteq 8.044$ $\qquad$ $t \doteq -0.044$

but $t > 0$, t is positive

∴ the ball hits the ground after 8.0 seconds.

Apr 25-3:03 PM

Ex.3 The size of a television screen or computer monitor is usually stated as the length of the diagonal. A screen has a 38-cm diagonal. The width of the screen is 6 cm more than the height. Find the dimensions of the screen to the nearest tenth.



$(x+6)^2 + x^2 = 38^2$

$(x+6)(x+6) + x^2 = 1444$

$x^2 + 6x + 6x + 36 + x^2 = 1444$

$2x^2 + 12x + 36 - 1444 = 0$

$2x^2 + 12x - 1408 = 0$

$\frac{2(x^2+6x-704)}{2} = \frac{0}{2}$ $\qquad \frac{2x}{2} = x$

$x^2 + 6x - 704 = 0$

QF: $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{-6 \pm \sqrt{(6)^2-4(1)(-704)}}{2(1)}$

$x = \frac{-6 \pm \sqrt{36+2816}}{2}$

$x = \frac{-6 \pm \sqrt{2852}}{2}$

$x = \frac{-6+\sqrt{2852}}{2}$ $\qquad x = \frac{-6-\sqrt{2852}}{2}$

$x \doteq 23.7$ $\qquad x \doteq -29.7$

x is height, $x > 0$

$\therefore$ height is 23.7cm

width is 29.7cm

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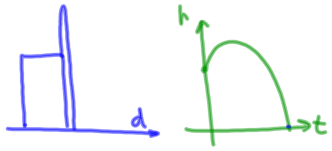
Assigned Work:

p. 357 #2, 3, 5, 7, 9, 14

Apr 25-2:54 PM

2. $h = 15 + 22t - 5t^2$



(a) y-int, set $t = 0$

$h = 15$

(b) 10m above ground,

set $h = 10$

$10 = 15 + 22t - 5t^2$

solve!

$0 = 5 + 22t - 5t^2$

$0 = -5t^2 + 22t + 5$

$0 = 5t^2 - 22t - 5$

S -22
P -25
I -25

QF:

(c) hit the ground, $h = 0$.

$0 = 15 + 22t - 5t^2$

$0 = -5t^2 + 22t + 15$

$0 = 5t^2 - 22t - 15$

S -22
P -75
I -25, 3

factor by decomposition

(d) use zeroes from (c)

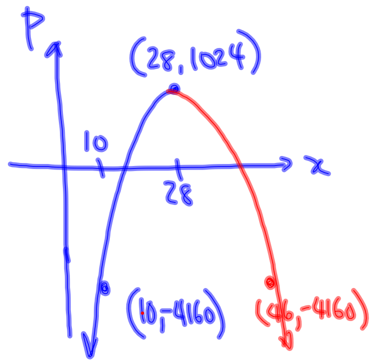
$t_{MP} = \frac{x_1 + x_2}{2}$

Sub t_{mp} into equation

Dec 1-10:34 AM

5. max:

	profit	price
	$p = 1024$	$x = 28$
	$p = -4160$	$x = 10$



$p = a(x - 28)^2 + 1024$

Sub $(10, -4160)$ OR $(46, -4160)$

to find a

(b) "break even" → profit = 0

set $p = 0$ and solve for x

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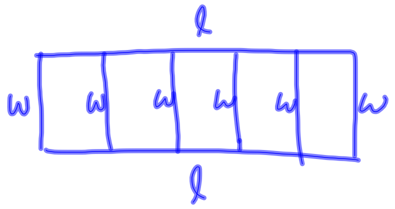
7.



$\frac{2l}{2} + \frac{6w}{2} = \frac{30}{2}$

$A = l \times w$ ② $\rightarrow$ maximum

$l + 3w = 15$

$l = 15 - 3w$ ①

Sub ① into ②

$A = (15-3w)(w) \rightarrow$ max $\rightarrow$ vertex

① find zeroes, set $A=0$ & solve

② find a.o.s. & sub into equation

⋮

$\therefore$ dimensions are $w = 2.5$ m

$l = 7.5$ m

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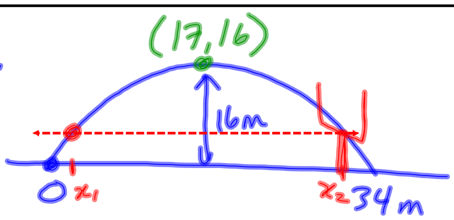
14.



(a)

Vertex

$y = a(x-17)^2 + 16$

sub $(34,0)$

zeroes

$y = a(x-0)(x-34)$

$y = ax(x-34)$

sub $(17,16)$

(b) set $h = 3.3$, solve for x

$\rightarrow$ 2 positive values

$\rightarrow$ want largest

(furthest from Maggie)

Dec 1-10:56 AM