In non-right triangles we cannot use the primary trigonometric ratio; there is no 90° angle, so there is no hypotenuse!

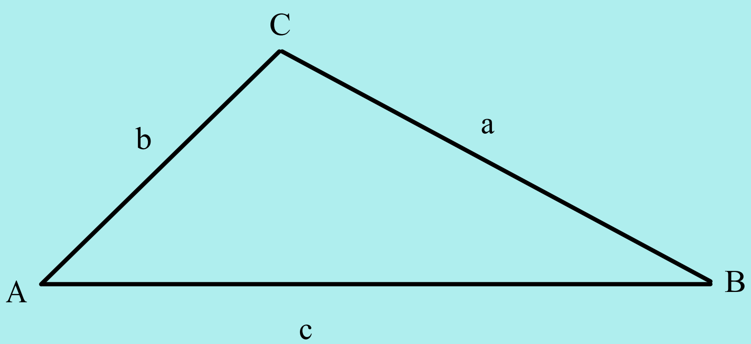
However, there still exists relationships between the sides and the angles in the triangle.

The relationships can be expressed in terms of sine or cosine and are called the Sine Law and the Cosine Law.

We will study these laws over the next few days.

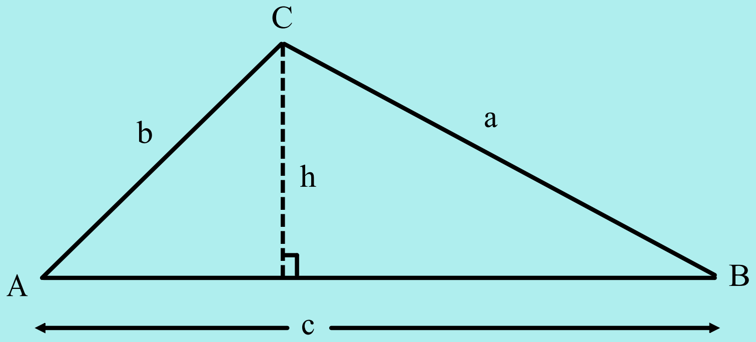
May 13-1:31 PM

Proving the Cosine Law:



May 14 - 9:32 PM

Proving the Cosine Law:



We can always create right triangles by drawing an altitude from any vertex.

Using trigonometry on each right triangle, we can relate the angles and sides of the overall triangle.

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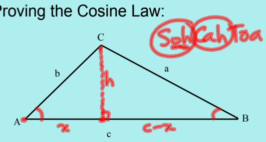
Proving the Cosine Law:

SohCahToa



$\cos A = \frac{x}{b}$ $\cos B = \frac{c-x}{a}$

$b^2 = x^2 + h^2$ $a^2 = (c-x)^2 + h^2$

$b^2 - x^2 = h^2$ ① $a^2 - (c-x)^2 = h^2$ ②

$$b^2 - x^2 = a^2 - (c-x)^2$$

$$b^2 - x^2 = a^2 - (c-x)(c-x)$$

$$b^2 - x^2 = a^2 - (c^2 - cx - cx + x^2)$$

$$b^2 - x^2 = a^2 - (c^2 - 2cx + x^2)$$

$$b^2 - x^2 = a^2 - c^2 + 2cx - x^2$$

$+x^2$ $+x^2$

$$b^2 = a^2 - c^2 + 2cx$$

$\cos A = \frac{x}{b}$

$b\cos A = x$

$$b^2 = a^2 - c^2 + 2c(b\cos A)$$

$$b^2 = a^2 - c^2 + 2bc\cos A$$

$$-a^2 = -b^2 - c^2 + 2bc\cos A$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

different altitude $b^2 = a^2 + c^2 - 2ac\cos B$

diff. alt. $c^2 = a^2 + b^2 - 2ab\cos C$

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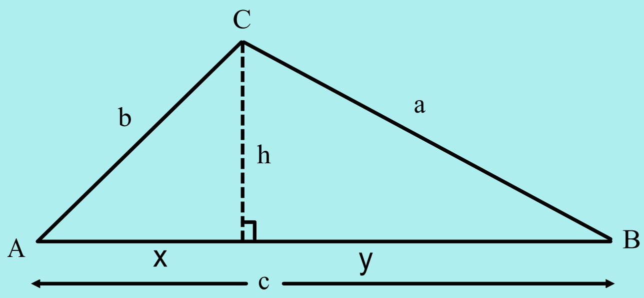

so that x + y = c

then y = c - x

$$x^2 + h^2 = b^2$$
$$h^2 = b^2 - x^2$$

$$y^2 + h^2 = a^2$$
$$h^2 = a^2 - y^2$$

$$set \quad h^2 = h^2$$
$$a^2 - y^2 = b^2 - x^2$$
$$a^2 = b^2 - x^2 + y^2$$
$$sub. \; y = c - x$$
$$a^2 = b^2 - x^2 + (c - x)^2$$
$$a^2 = b^2 - x^2 + c^2 - 2cx + x^2$$
$$a^2 = b^2 + c^2 - 2cx$$
$$sub. \; x = b\cos A$$
$$a^2 = b^2 + c^2 - 2cb\cos A$$

we will also need:

$$\cos\theta = \frac{adj}{hyp}$$
$$\cos A = \frac{x}{b}$$
$$b\cos A = x$$

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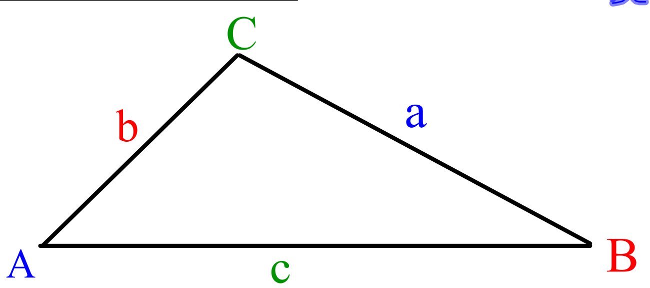
The Cosine Law

Dec 15/2011



The Cosine Law (2 formats) for Δ ABC:

$$a^2 = b^2 + c^2 - 2bc\cos A$$

or

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

You decide which format to use depending on what you are solving for.

May 15-2:45 PM

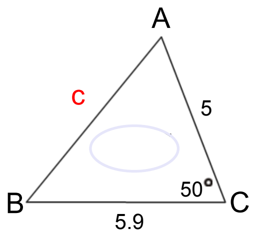
The Cosine Law can be re-written for the other sides and angles of the triangle:

$$b^2 = a^2 + c^2 - 2ac\cos B \qquad \cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$c^2 = a^2 + b^2 - 2ab\cos C \qquad \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

May 13-3:44 PM

Ex. 1 Find the length of side c.



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$c^2 = (5.9)^2 + (5)^2 - 2(5.9)(5)\cos 50°$$

$$c^2 \doteq 34.81 + 25 - 59(0.6428)$$

$$c^2 \doteq 59.81 - 37.9252$$

$$c^2 \doteq 21.8848$$

$$c \doteq \pm 4.6781 \quad , \; c > 0$$

$$\boxed{c \doteq 4.7}$$

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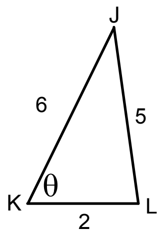
Ex.2 Solve for θ.



$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos K = \frac{j^2 + l^2 - k^2}{2jl}$$

$$\cos\theta = \frac{(2)^2 + (6)^2 - (5)^2}{2(2)(6)}$$

$$\cos\theta = \frac{4 + 36 - 25}{24}$$

$$\cos\theta = \frac{15}{24}$$

$$\cos\theta = 0.625$$

$$\theta = \cos^{-1}(0.625)$$

$$\boxed{\theta \doteq 51.3^\circ}$$

Dec 13-10:20 PM

Assigned Work:

p.438 # 2, 3b

p.443 # 2, 3, 4, 5ac

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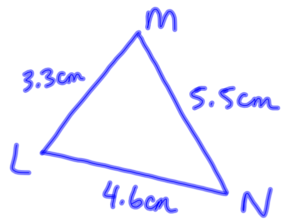
p 438

3(b) $\cos A = \dfrac{b^2+c^2-a^2}{2bc}$

$$\cos P = \frac{q^2+r^2-p^2}{2qr}$$

Dec 16-10:30 AM

p. 443 # 5(c)



$$\cos L = \frac{m^2+n^2-l^2}{2mn}$$

$$\cos L = \frac{(4.6)^2+(3.3)^2-(5.5)^2}{2(4.6)(3.3)}$$

$$\cos L = \frac{1.8}{30.36}$$

$$L \doteq \cos^{-1}(0.0593)$$

$$L \doteq 86.6^\circ$$

$$\cos M = \frac{l^2+n^2-m^2}{2ln}$$

Dec 16-10:32 AM