Unit 1 - Quadratic Functions & Relations

Finding Max/Min Values by Completing the Square

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Finding Max/Min Values by Completing the Square

Feb 7/2012

Standard Form $y = ax^2 + bx + c$ $\xrightarrow{\text{complete the square}}$ Vertex Form $y = a(x-h)^2 + k$

Vertex is (h, k)

a > 0: opens up (has a minimum)
a < 0: opens down (has a maximum)

k is the optimal value (max or min value)

h is the x-value where the max/min occurs
x = h is the equation of the axis of symmetry

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Completing the Square Using Fractions:

Ex.1 Complete the square:

(a) $y = 3x^2 + 2x - 11$

$y = 3\left[\frac{3x^2}{3} + \frac{2x}{3}\right] - 11$

$y = 3\left[x^2 + \frac{2}{3}x\right] - 11$

$\frac{2}{3} \div \frac{2}{1} = \frac{2}{3} \cdot \frac{1}{2}$

$= \frac{2}{6}$

$= \frac{1}{3}$

$\left(\frac{1}{3}\right)^2 = \frac{1^2}{3^2} = \frac{1}{9}$

$y = 3\left[\underbrace{x^2 + \frac{2}{3}x + \frac{1}{9}}_{\left(x+\frac{1}{3}\right)^2} - \frac{1}{9}\right] - 11$

$y = 3\left[\left(x+\frac{1}{3}\right)^2 - \frac{1}{9}\right] - 11$

$y = 3\left(x+\frac{1}{3}\right)^2 - \frac{3}{9} - 11$

$y = 3\left(x+\frac{1}{3}\right)^2 - \frac{1}{3} - \frac{33}{3}$

$y = 3\left(x+\frac{1}{3}\right)^2 - \frac{34}{3}$

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(b) $y = \underbrace{5x^2 + 8x}_{} - 3$

$y = 5\left[x^2 + \frac{8}{5}x\right] - 3$

$\frac{8}{5} \cdot \frac{1}{2} = \frac{8}{10} = \frac{4}{5}$

$\left(\frac{4}{5}\right)^2 = \frac{16}{25}$

$y = 5\left[\underbrace{x^2 + \frac{8}{5}x + \frac{16}{25}}_{} - \frac{16}{25}\right] - 3$

$5\left(-\frac{16}{25}\right)$

$= -\frac{\cancel{80}\ 16}{\cancel{25}\ 5}$

$y = 5\left[\left(x+\frac{4}{5}\right)^2 - \frac{16}{25}\right] - 3$

$y = 5\left(x+\frac{4}{5}\right)^2 - \frac{16}{5} - \frac{15}{5}$

$y = 5\left(x+\frac{4}{5}\right)^2 - \frac{31}{5}$

$V\left(-\frac{4}{5}, -\frac{31}{5}\right)$ opens up

min. value $= -\frac{31}{5}$

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Ex.2 Find the optimal value of $y = 5x - 3x^2 + 4$

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It is permissible, and sometimes preferable, to use terminating decimals (i.e., exact values):

Ex.3 Find the optimal value of $y = -20x^2 + 180x + 4400$

$y = -20x^2 + 180x + 4400$ $\quad -\frac{9}{2} = -4.5$

$y = -20[x^2 - 9x] + 4400$ $\quad (-4.5)^2 = 20.25$

$y = -20[x^2 - 9x + 20.25 - 20.25] + 4400$

$y = -20[(x-4.5)^2 - 20.25] + 4400$

$y = -20(x-4.5)^2 + 405 + 4400$

$y = -20(x-4.5)^2 + 4805$

V(4.5, 4805)

opens down

optimal value is a max. 4805

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Homework:

p.115 # 1cegik, 3odd, 7, 14, 16
e

stevesweeney.pbworks.com

$$3 \div \left(-\frac{1}{2}\right) = 3 \times \left(-\frac{2}{1}\right)$$
$$= -\frac{6}{1}$$
$$= -6$$

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3(e)

$$y = -\frac{1}{3}x^2 + 2x + 4$$
$$y = -\frac{1}{3}(x^2 - 6x) + 4$$
$$y = -\frac{1}{3}\left[x^2 - 6x + 9 - 9\right] + 4$$
$$y = -\frac{1}{3}\left[(x-3)^2 - 9\right] + 4$$
$$y = -\frac{1}{3}(x-3)^2 + 3 + 4$$
$$y = -\frac{1}{3}(x-3)^2 + 7$$

$V(3,7)$, opens down.

max. of 7

$$\frac{-\frac{1}{3}x^2}{-\frac{1}{3}} = x^2$$
$$\frac{2x}{-\frac{1}{3}} = 2x\left(-\frac{3}{1}\right)$$
$$= -6x$$
$$\frac{-6}{2} = -3$$
$$(-3)^2 = 9$$

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7. any number → x

$y = x^2 - 8x + 35$

↑

result.

what is smallest possible value for y?

$y = x^2 - 8x + 16 - 16 + 35$

$y = (x-4)^2 + 19$

$V(4,19)$, min value is 19

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14.

$h = -\frac{1}{2}gt^2 + v_0t + h_0$

g ↑ 9.8, v_0 ↑ 34.3, h_0 ↑ 2.1

$h = -4.9t^2 + 34.3t + 2.1$

$h = -4.9\left[t^2 - 7t\right] + 2.1$

$h = -4.9\left[t^2 - 7t + 12.25 - 12.25\right] + 2.1$

$h = -4.9\left[(t-3.5)^2 - 12.25\right] + 2.1$

$h = -4.9(t-3.5)^2 + 62.125$

∴ max height of 62.125 m after 3.5 seconds.

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