Finding Max/Min Values Using Zeroes

Feb 8/2012

Our motivation for completing the square is generally to find the vertex of the quadratic relation.

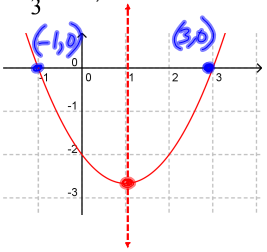
It is also possible to find the vertex from the zeroes, or roots, of the quadratic, and the fact that the parabola is symmetric about the vertex.

In other words, the axis of symmetry is half-way between the zeroes, and it matches the x-coordinate of the vertex.

Sub this value into the equation to find the y-coordinate of the vertex, or the optimal value.

Feb 6-3:52 PM

Ex.1 Given the graph of $y=\frac{2}{3}x^2-\frac{4}{3}x-2$, determine the minimum value.



zeroes: $(-1,0)$ and $(3,0)$

Axis of symmetry is MP of zeroes

$x=\frac{(-1)+(3)}{2}$

$x=1 \rightarrow$ also x_{vertex}

Sub $x=1$ into equation to find y_{vertex}

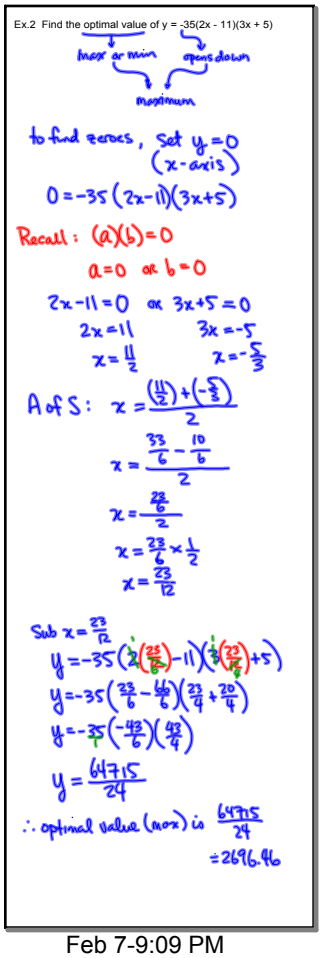
$y=\frac{2}{3}(1)^2-\frac{4}{3}(1)-2$

$y=\frac{2}{3}-\frac{4}{3}-\frac{6}{3}$

$y=-\frac{8}{3}$

$\therefore V\left(1,-\frac{8}{3}\right)$, min. value is $-\frac{8}{3}$

Feb 7-8:57 PM



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Ex.3 The city transit system carries 24,800 bus riders per day for a fare of $3.15. The city hopes to reduce car pollution by getting more people to ride the bus, while maximizing the transit system's revenue at the same time. A survey indicates that the number of riders will increase by 800 for every $0.05 decrease in the fare. What fare will produce the greatest revenue?

money taken in

Revenue = (# riders)(ticket cost)

$R = (24800)(3.15)$

$R = (24800 + 800)(3.15 - 0.05)$

$R = (24800 + 2(800))(3.15 - 2(0.05))$

$R = (24800 + 800x)(3.15 - 0.05x)$

① find zeroes → R = 0

② find AofS → MP of zeroes

③ sub x into equation

Feb 7-9:20 PM

Ex.4 Find the optimal value of $y = -20x^2 + 180x + 4400$

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Homework:

p.115 # 2f, 3bcfgk, 9, 13, 15, 18

Feb 1-7:30 PM

3(b)

$y = x^2 - x - 2$ S -1, P -2, I -2, 1

$y = x^2 - 2x + x - 2$

$y = x(x-2) + 1(x-2)$

$y = (x-2)(x+1)$

for zeroes, set $y = 0$

$0 = (x-2)(x+1)$

$x - 2 = 0$ or $x + 1 = 0$

$\boxed{x = 2}$ $\boxed{x = -1}$

$x_v = \frac{2 + (-1)}{2}$

$= \frac{1}{2}$

Sub $x = \frac{1}{2}$

$y = \left(\frac{1}{2} - 2\right)\left(\frac{1}{2} + 1\right)$

$y = \left(-\frac{3}{2}\right)\left(\frac{3}{2}\right)$

$y = -\frac{9}{4}$ → min value is $-\frac{9}{4}$

Feb 9-1:53 PM

9. $R = (600)(15)$

$R = (600 - 30)(15 + 1)$

$R = (600 - 2(30))(15 + 2(1))$

$R = (600 - 30x)(15 + x)$

Let x be the number of $1 price increases

Set $R = 0$

$0 = (600 - 30x)(15 + x)$

$600 - 30x = 0$ or $15 + x = 0$

$\frac{600}{30} = \frac{30x}{30}$ $\boxed{x = -15}$

$\boxed{x = 20}$

$x_v = \frac{20 + (-15)}{2}$

$= 2.5$

sub $x = 2.5 \longrightarrow y_v$

$y_v = 9187.5$

$V(2.5, 9187.5)$

new price, max. revenue

$= 15 + 2.5$

$= 17.50$

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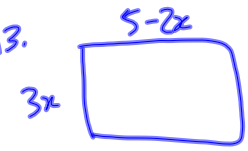
13.



$A = 3x(5-2x)$

for zeroes, set $A=0$

$0 = 3x(5-2x)$

$3x=0$ or $5-2x=0$

$x=0$ $\quad 5=2x$

$x=2.5$

$x_v = \frac{0+2.5}{2}$

$= 1.25$

$A_{max} = 3(1.25)(5-2(1.25))$

$= 3.75(2.5)$

$= 9.375$

Feb 9-2:03 PM

18. $h = -0.043d^2 + 2.365d$

$h = d(-0.043d + 2.365)$

Set $h=0$

$0 = d(-0.043d + 2.365)$

$d=0$ $\quad -0.043d + 2.365 = 0$

$-0.043d = -2.365$

$d = 55$

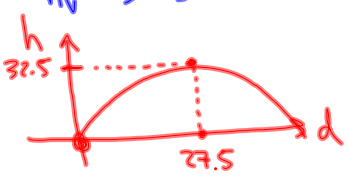
$d_v = \frac{0+55}{2}$

$= 27.5$

sub $d=27.5$

$h_v = -0.043(27.5)^2 + 2.365(27.5)$

$h_v = 32.5$



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