Recall: The simplest quadratic relation is $y = x^2$

On rearranging, it is possible to get answers
in the form $x = \pm\sqrt{y}$

With actual values, we might see results such as

$$\sqrt{5} \qquad 3\sqrt{2} \qquad \frac{\sqrt{3}}{2}$$

It is often required to keep answers in this exact form.

Feb 6-3:52 PM

# Working With Radicals

Feb9/2012

index understood to be 2

index of 3

$$\sqrt{5} \qquad \text{radical sign} \qquad \sqrt[3]{5}$$

radicand

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## Multiplying & Dividing Radicals

In general, $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

and $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$ *where* $b \neq 0$

Ex.1

~~(a) $\sqrt{27}$~~

(a) $\sqrt{3}\sqrt{9}$

$= \sqrt{3 \cdot 9}$

$= \sqrt{27}$

(b) $\sqrt{\frac{16}{9}}$

$= \frac{\sqrt{16}}{\sqrt{9}}$

$= \frac{4}{3}$

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## Simplifying Radicals

A radical is in its simplest form when:

- the radicand has no perfect square factors (other than 1) $\sqrt{8} = \sqrt{4}\sqrt{2} = 2\sqrt{2}$

- the radicand contains no fractions $\sqrt{\frac{1}{4}} = \frac{1}{2}$

- no radical appears in the denominator $\frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

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Ex.2 Simplify

$32 = 2\cdot 2\,2\cdot 2\cdot 2$

(a) $\sqrt{32}$

$= \sqrt{2\cdot 16}$

$= 4\sqrt{2}$

(b) $2\sqrt{75}$

$= 2\sqrt{3\cdot 25}$

$= 2\sqrt{3}\sqrt{25}$

$= 10\sqrt{3}$

(c) $-3\sqrt{8}$

$= -3\sqrt{4\cdot 2}$

$= -3\sqrt{4}\sqrt{2}$

$= -6\sqrt{2}$

(d) $\frac{1}{2}\sqrt{\frac{72}{25}}$

$= \frac{1}{2}\frac{\sqrt{72}}{\sqrt{25}}$

$= \frac{1}{2}\cdot\frac{\sqrt{36\cdot 2}}{5}$

$= \frac{\overset{3}{\cancel{6}}\sqrt{2}}{\cancel{10}\,5}$

$= \frac{3\sqrt{2}}{5}$

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Homework:

p.106 # (1 - 4)(odd)

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## Adding & Subtracting Radicals

To add or subtract radicals, they must have the same radicand. It is advisable to simplify radicals to ensure all like terms (same radicand) are revealed.

Ex.3 Simplify

(a) $4\sqrt{3}-2\sqrt{5}+6\sqrt{3}+5\sqrt{5}$

$= 10\sqrt{3}+3\sqrt{5}$

$4x-2y+6x+5y$

$= 10x+3y$

(b) $2\sqrt{12}-5\sqrt{27}+3\sqrt{48}$

$= 2\sqrt{4\cdot 3}-5\sqrt{9\cdot 3}+3\sqrt{16\cdot 3}$

2 3 4

$= 4\sqrt{3}-15\sqrt{3}+12\sqrt{3}$

$4\cdot 3$

$= \sqrt{3}$

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## Binomial Multiplication of Radicals

Recall: $(a+b)(c+d) = ac+ad+bc+bd$

Ex.4 Expand & Simplify

$\left(3\sqrt{5}+2\right)\left(2\sqrt{5}-3\right)$

$= (3\sqrt{5})(2\sqrt{5})+(3\sqrt{5})(-3)+(2)(2\sqrt{5})+(2)(-3)$

$= 6\sqrt{25}-9\sqrt{5}+4\sqrt{5}-6$

$6(5)$

$= 30-9\sqrt{5}+4\sqrt{5}-6$

$= 24-5\sqrt{5}$

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Rationalizing the Denominator

A radical is not permitted in the denominator. If the denominator is a binomial, multiply by the conjugate of the denominator.

Given $a\sqrt{b}+c\sqrt{d}$, the conjugate would be $a\sqrt{b}-c\sqrt{d}$

Given $a\sqrt{b}-c\sqrt{d}$, the conjugate would be $a\sqrt{b}+c\sqrt{d}$

Ex.5 Find the conjugate of each radical

(a) $\sqrt{5}-\sqrt{2}$

$\sqrt{5}+\sqrt{2}$

(b) $3\sqrt{5}+2\sqrt{10}$

$3\sqrt{5}-2\sqrt{10}$

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Ex.6 Rationalize the denominator

$$\frac{4\sqrt{3}-2\sqrt{2}}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$$

← expand

← expand

$$= \frac{(4\sqrt{3})(\sqrt{3})+(4\sqrt{3})(\sqrt{2})+(-2\sqrt{2})(\sqrt{3})+(-2\sqrt{2})(\sqrt{2})}{(\sqrt{3})(\sqrt{3})+(\sqrt{3})(\sqrt{2})+(-\sqrt{2})(\sqrt{3})+(-\sqrt{2})(\sqrt{2})}$$

$$= \frac{4\sqrt{9}+4\sqrt{6}-2\sqrt{6}-2\sqrt{4}}{\sqrt{9}+\sqrt{6}-\sqrt{6}-\sqrt{4}}$$

$$= \frac{12+2\sqrt{6}-4}{3-2}$$

$$= \frac{8+2\sqrt{6}}{1}$$

$$= 8+2\sqrt{6}$$

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Homework: 4c

p.106 # (1 - 4)(odd) 2?

p.139 # (1 - 7)(odd) 7?, 1?, 4?

6e

Feb 1-7:30 PM

p.106

#2(j) $\frac{12\sqrt{75}}{4\sqrt{3}} = \frac{12\sqrt{25\cdot 3}}{4\sqrt{3}}$

$= \frac{\overset{3}{\cancel{12}}(5)\overset{1}{\cancel{\sqrt{3}}}}{\underset{1}{\cancel{4}}\underset{1}{\cancel{\sqrt{3}}}}$

$= 15$

Feb 10-1:39 PM

p. 106

\# 4(c) $\frac{6+\sqrt{8}}{2}$

$$= \frac{6+\sqrt{4\cdot 2}}{2}$$

$$= \frac{6+2\sqrt{2}}{2}$$

$$= \frac{\overset{3}{\cancel{6}}+\overset{1}{\cancel{2}}\sqrt{2}}{\underset{1}{\cancel{2}}}$$

$$= 3+\sqrt{2}$$

$$= \frac{6+2\sqrt{2}}{2}$$

$$= \frac{\cancel{2}(3+\sqrt{2})}{\cancel{2}}$$

$$= 3+\sqrt{2}$$

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p. 139 # 7

(j) $$\frac{(2\sqrt{7}+\sqrt{5})}{(3\sqrt{7}-2\sqrt{5})} \times \frac{(3\sqrt{7}+2\sqrt{5})}{(3\sqrt{7}+2\sqrt{5})}$$

$$= \frac{6(7)+4\sqrt{35}+3\sqrt{35}+2(5)}{9(7)+6\sqrt{35}-6\sqrt{35}-4(5)}$$

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