Solving Quadratic Equations

Feb 10/2012

Recall: To solve is to find the value(s) that satisfy the equation, or make it true.

We often use this technique to find the zeroes, or roots, of a quadratic relation.

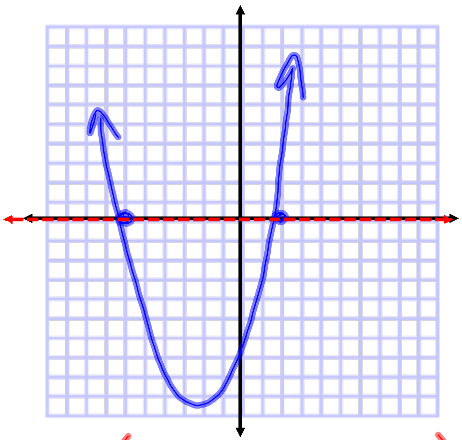
Ex.1 Solve $\underbrace{x^2 + 4x - 12}_{} = \underbrace{0}_{}$

$y = x^2 + 4x - 12$

$y = (x+6)(x-2)$

zeroes: $-6, 2$

$y = 0$ (horizontal line)



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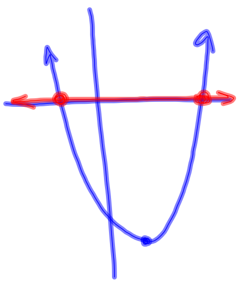
Factoring is not the only option:

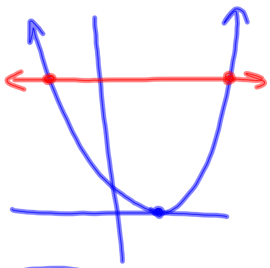
Ex.2 Solve $x^2 - 6x - 27 = 0$ by completing the square.

$\underbrace{x^2 - 6x + 9} - 9 - 27 = 0$

$(x-3)^2 - 36 = 0$

$(x-3)^2 = 36$





$x - 3 = \pm\sqrt{36}$

$x - 3 = \pm 6$

$x = 3 \pm 6$

$x = 9 \quad x = -3$

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Ex.3 Solve $2x^2 - 5x - 1 = 0$ using by completing the square.

$$2x^2 - 5x - 1 = 0$$
$$2\left(x^2 - \frac{5}{2}x\right) - 1 = 0$$
$$2\left(x^2 - \frac{5}{2}x + \frac{25}{16} - \frac{25}{16}\right) - 1 = 0$$
$$2\left[\left(x - \frac{5}{4}\right)^2 - \frac{25}{16}\right] - 1 = 0$$
$$2\left(x - \frac{5}{4}\right)^2 - \frac{25}{8} - \frac{8}{8} = 0$$
$$2\left(x - \frac{5}{4}\right)^2 - \frac{33}{8} = 0$$
$$\frac{1}{2} \times 2\left(x - \frac{5}{4}\right)^2 = \frac{33}{8} \times \frac{1}{2}$$
$$\left(x - \frac{5}{4}\right)^2 = \frac{33}{16}$$
$$x - \frac{5}{4} = \pm\sqrt{\frac{33}{16}}$$

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

$$x = \frac{5}{4} \pm \frac{\sqrt{33}}{4}$$
$$x = \frac{5 \pm \sqrt{33}}{4}$$
$$x = \frac{5 + \sqrt{33}}{4} \quad \text{or} \quad x = \frac{5 - \sqrt{33}}{4}$$

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Deriving the Quadratic Formula

$$ax^2 + bx + c = 0$$
$$a\left[x^2 + \frac{b}{a}x\right] + c = 0$$
$$a\left[x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2\right] + c = 0$$
$$a\left[\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2}\right] = -c$$
$$\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} = -\frac{c}{a}$$
$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$$
$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{4ac}{4a^2}$$
$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$
$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

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Recall: The general quadratic formula

Given $ax^2 + bx + c = 0$, the solutions are:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The number of real solutions can be determined from the discriminant, $b^2 - 4ac$:

$b^2 - 4ac > 0$	two real solutions
$b^2 - 4ac = 0$	one real solution
$b^2 - 4ac < 0$	no real solutions

Note: The GQF is derived by completing the square on $ax^2 + bx + c = 0$. They are essentially the same.

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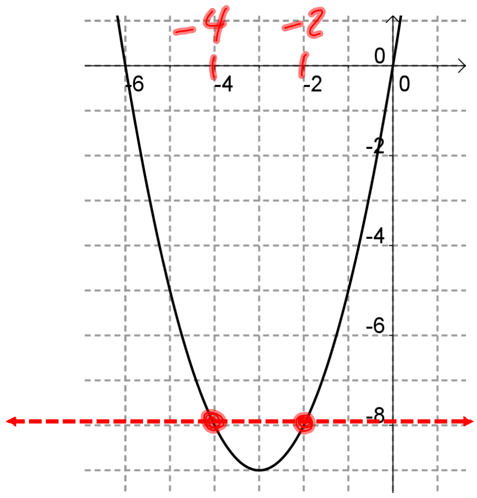
Given the relation $y = ax^2 + bx + c$, any of these methods can be used to solve for any value of y.

Consider $y = x^2 + 6x$, and solve for $y = -8$.



$x^2 + 6x = -8$

$x^2 + 6x + 8 = 0$

$(x+2)(x+4) = 0$

$x+2=0$ OR $x+4=0$

$x=-2$ $x=-4$

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Assigned Work:

p.128 # 2~~odd~~ ace, 3ace, 4ace, 12ac, 13~~odd~~ ace
p.130 # 17, ~~21~~, 23, ~~28ab~~, 29

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p. 128 # 3(e)

$$\left(\frac{-7}{2}\right)^2 = \frac{49}{4}$$

$$d^2 = 7d - 9$$

$$d^2 - 7d + 9 = 0$$

$$\underbrace{d^2 - 7d + \frac{49}{4}} - \frac{49}{4} + 9 = 0$$

$$\left(d - \frac{7}{2}\right)^2 - \frac{49}{4} + \frac{36}{4} = 0$$

$$\left(d - \frac{7}{2}\right)^2 - \frac{13}{4} = 0$$

$$\left(d - \frac{7}{2}\right)^2 = \frac{13}{4}$$

$$d - \frac{7}{2} = \pm\sqrt{\frac{13}{4}}$$

$$d = \frac{7}{2} \pm \frac{\sqrt{13}}{2}$$

$$d = \frac{7 \pm \sqrt{13}}{2}$$

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4(c)

$$6x^2+3x-2=0$$
$$6\left(x^2+\frac{3}{6}x\right)-2=0$$
$$6\left[x^2+\frac{1}{2}x+\frac{1}{16}-\frac{1}{16}\right]-2=0$$
$$6\left[\left(x+\frac{1}{4}\right)^2-\frac{1}{16}\right]-2=0$$
$$6\left(x+\frac{1}{4}\right)^2-\frac{6}{16}-\frac{32}{16}=0$$
$$6\left(x+\frac{1}{4}\right)^2-\frac{38}{16}=0$$
$$6\left(x+\frac{1}{4}\right)^2=\frac{19}{8}$$
$$\left(x+\frac{1}{4}\right)^2=\frac{19}{48}$$
$$x+\frac{1}{4}=\pm\sqrt{\frac{19}{48}}$$
$$x=-\frac{1}{4}\pm\frac{\sqrt{19}}{4\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$$
$$x=-\frac{1\times 3}{4\times 3}\pm\frac{\sqrt{57}}{12}$$
$$x=\frac{-3\pm\sqrt{57}}{12}$$

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12(a)

$$3x^2+6x+1=0$$
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-6\pm\sqrt{36-12}}{6}$$
$$x=\frac{-6\pm\sqrt{24}}{6}$$
$$x=\frac{-6\pm2\sqrt{6}}{6}$$
$$x=\frac{\cancel{2}^{1}(-3\pm\sqrt{6})}{\cancel{6}\,3}$$
$$x=\frac{-3\pm\sqrt{6}}{3}$$

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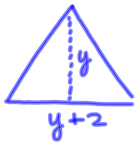
17.



$$A = \frac{1}{2}bh$$
$$5 = \frac{1}{2}(y+2)(y)$$
$$0 = \frac{1}{2}(y+2)(y) - 5$$
$$0 = \frac{1}{2}(y^2+2y) - 5$$
$$0 = \frac{1}{2}\left[y^2+2y+1-1\right] - 5$$
$$0 = \frac{1}{2}\left[(y+1)^2-1\right] - 5$$
$$0 = \frac{1}{2}(y+1)^2 - 0.5 - 5$$
$$5.5 = \frac{1}{2}(y+1)^2$$
$$11 = (y+1)^2$$
$$\pm\sqrt{11} = y+1$$
$$-1 \pm \sqrt{11} = y$$

decimals

$y =$ ______ $y =$ ______ negative is inadmissible

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23. p, q

$$p = 3q$$
$$(p+q) + pq = 224$$
$$(3q+q) + (3q)q = 224$$
$$4q + 3q^2 - 224 = 0$$
$$3q^2 + 4q - 224 = 0$$

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