Transformations of Functions

Feb 27/2012

Recall: In Gr.10, we explored transformations of a quadratic relation. For consistency, we use vertex form.

$$y = a(x-h)^2 + k$$

What are the possible transformations?
(read from left to right)

a → vertical reflection

a → vertical stretch / compression

h → horizontal translation / shift

k → vertical translation / shift

$$y = a(x-h)^2 + k$$

1&2 3 4

Summary of possible transformations (in order):

1. Vertical Reflection: a < 0

2. Vertical Scaling (stretch or compress):
   - stretch when a < -1 or a > 1 (or |a| > 1)
     - parabola is thinner
   - compress when -1 < a < 1 (or |a| < 1)
     - parabola is wider

3. Horizontal Translation (shift left or right) by h

4. Vertical Translation (shift up or down) by k

Using function notation,

$$y = x^2 \quad \text{becomes} \quad y = f(x)$$

so $y = a(x-h)^2 + k$ becomes $y = af(x-h) + k$

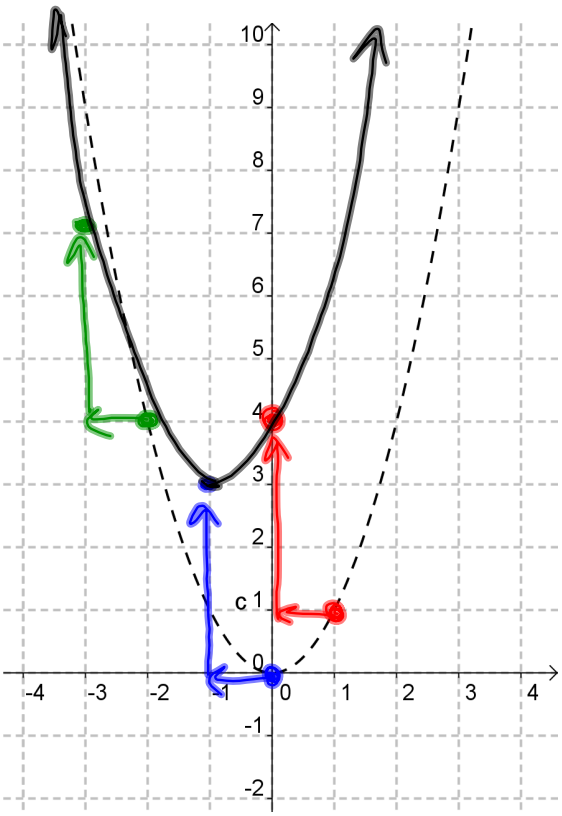
The transformations produced by a, h, and k can be applied to any function.

Ex.1 State the transformations, in the correct order, that $y = f(x)$ has undergone to obtain $y = f(x+1) + 3$

① shift left by 1

② shift up by 3

Using the previous example, what if the parent function is a quadratic? Then $f(x) = x^2$.
Sketch the transformed parabola on the grid below.
(Use a table of values if you want to)



$y = (x+1)^2 + 3$

$V(-1, 3)$

other "key points"

$(1,1) \rightarrow (0,4)$

$(-1,1) \rightarrow (-2,4)$

$(2,4) \rightarrow (1,7)$

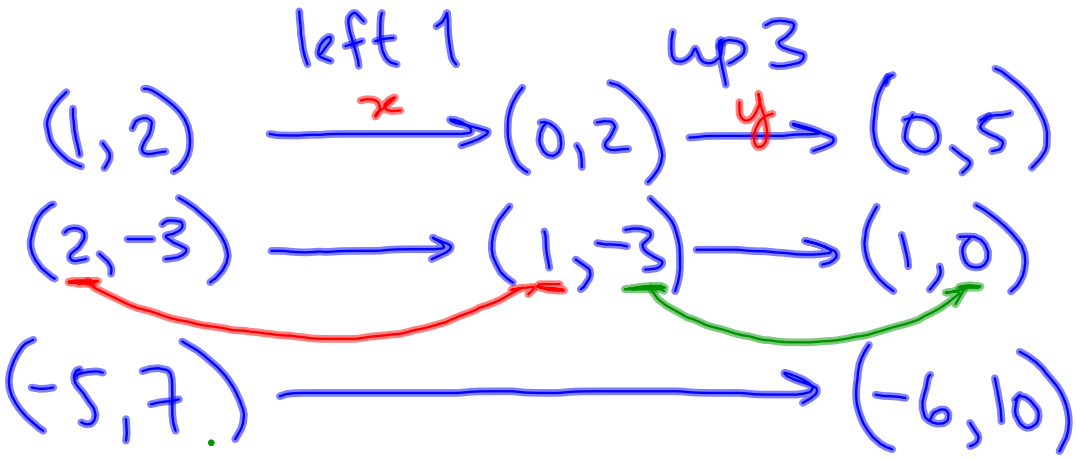
$(-2,4) \rightarrow (-3,7)$

What if the parent function is a set of ordered pairs?

If $f(x)=\{(1,\ 2),\ (2,\ -3),\ (-5,\ 7)\}$

what would $y=f(x+1)+3$ become?

(Use a table of values and/or graph if you want to!)



What if the parent function is the square root function?
Then $f(x)=\sqrt{x}$.
What would $y=f(x+1)+3$ become?
(Use a table of values if you want to)

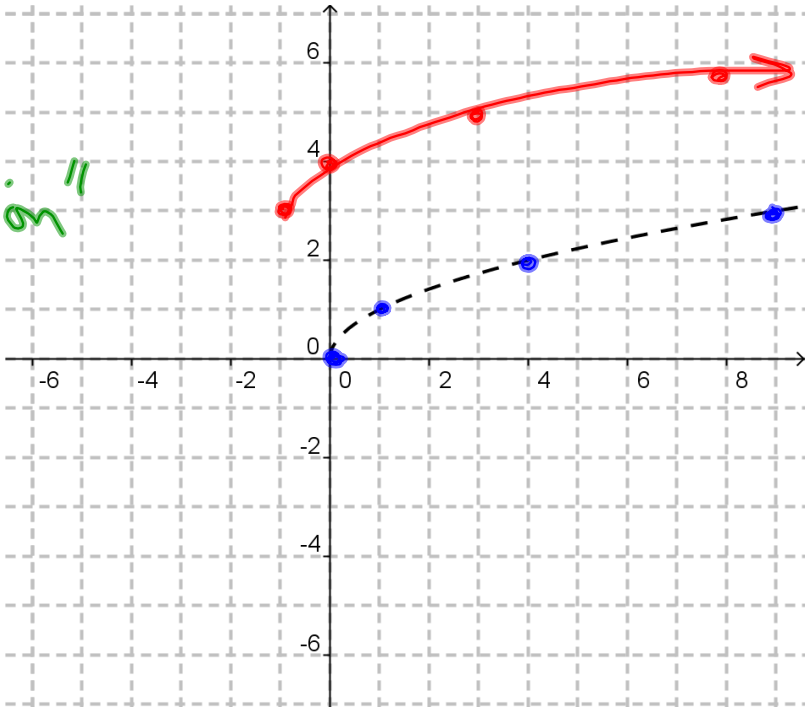
$y=\sqrt{x+1}+3$

① shift left 1

② shift up 3

"parent function"

x	$y=\sqrt{x}$
0	0
1	1
4	2
9	3



What if the parent function is the reciprocal function?

Then $f(x)=\frac{1}{x}$.

What would $y=f(x+1)+3$ become?

(Use a table of values if you want to)

asymptotes:

$y=0 \rightarrow y=3$

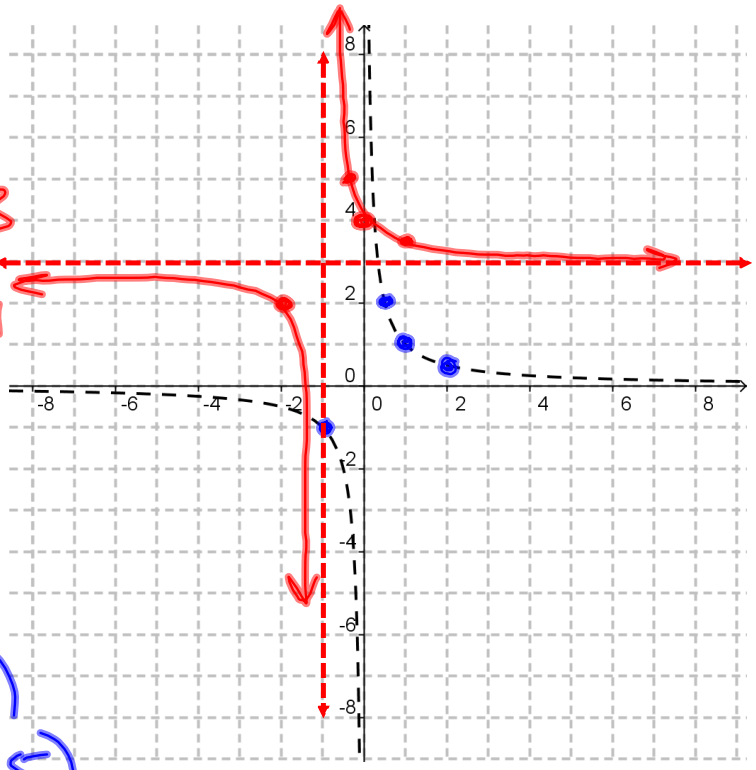
$x=0 \rightarrow x=-1$

$(1,1) \rightarrow (0,4)$

$(-1,-1) \rightarrow (-2,2)$

$(2,0.5) \rightarrow (1,3.5)$

$(0.5,2) \rightarrow (-0.5,5)$



Ex: Given $y = 2f(x) + 3$ describe the transformations and apply them to the square root function. $y = \sqrt{x}$

(Use a table of values and/or graph if you want to!)

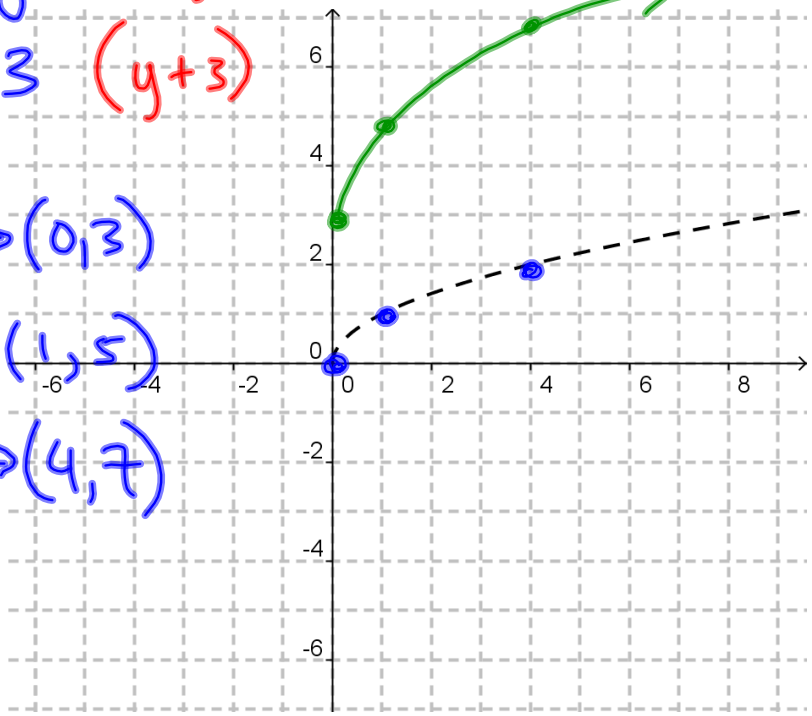
① v. stretch by 2 $(y \times 2)$

② v. shift up 3 $(y + 3)$

$(0,0) \to (0,0) \to (0,3)$

$(1,1) \to (1,2) \to (1,5)$

$(4,2) \to (4,4) \to (4,7)$



Ex: Given $y = 0.4f(x+1)$, describe the transformations and apply them to the reciprocal function.

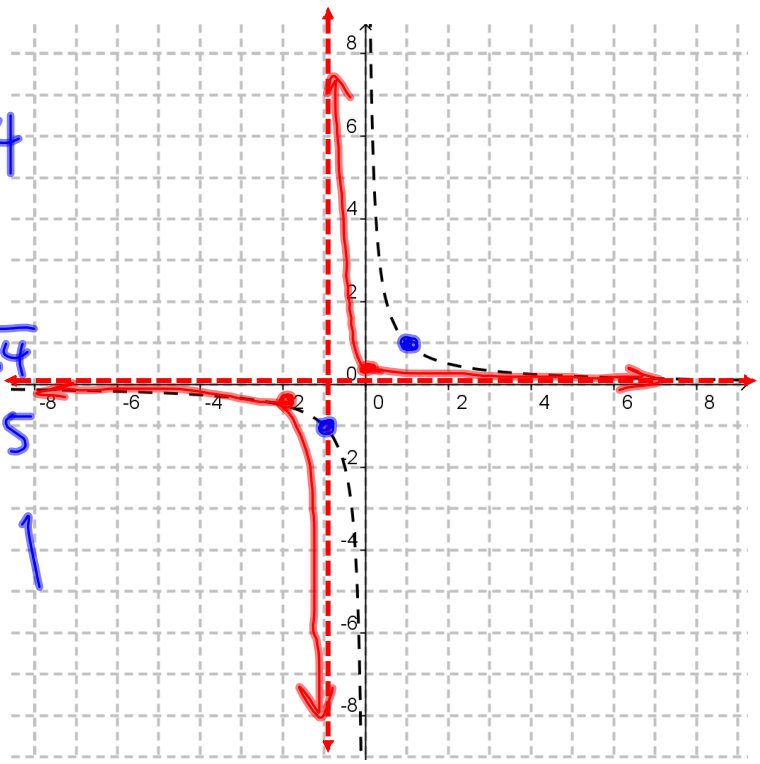
(Use a table of values and/or graph if you want to!)

① v. scaling by 0.4

or

v. compress by $\frac{1}{0.4} = 2.5$

② h. shift left 1



asymptotes:

$y = 0 \rightarrow y = 0$ HA does not change

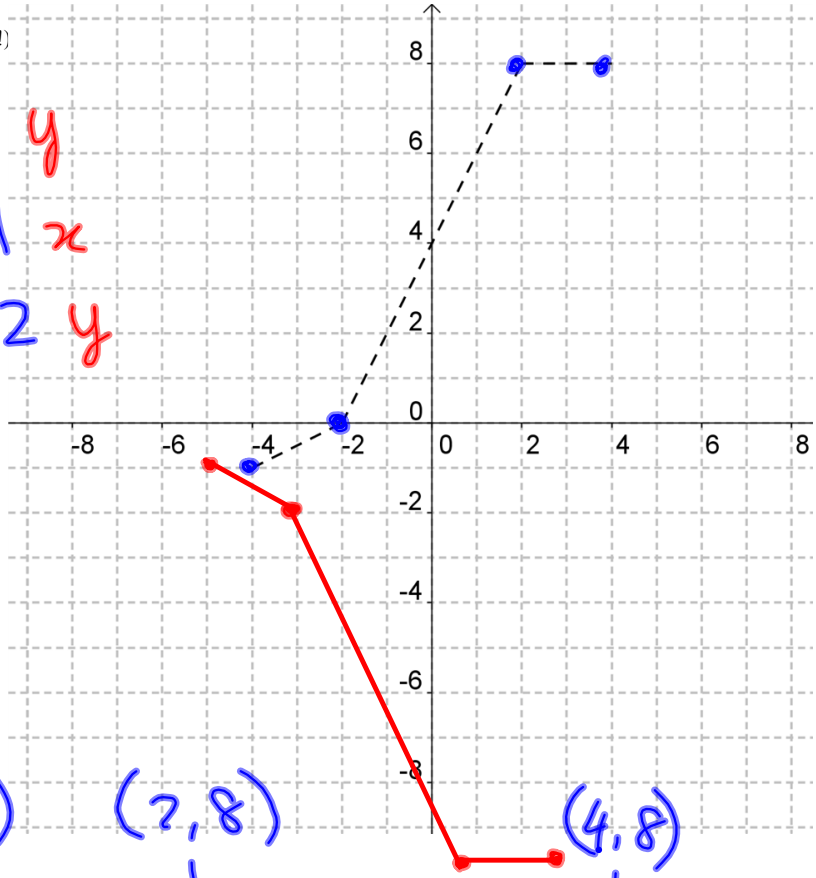
$x = 0 \rightarrow x = -1$ VA shift left 1

$(1,1) \rightarrow (0,0.4)$

$(-1,-1) \rightarrow (-2,-0.4)$

Ex: The graph on the right shows the function $y = f(x)$.
Given $y = -f(x+1) - 2$, describe the transformations and apply them to the function

(Use a table of values with the key points, if you want to!)



① v. reflection y

② h. shift left 1 x

③ v. shift down 2 y

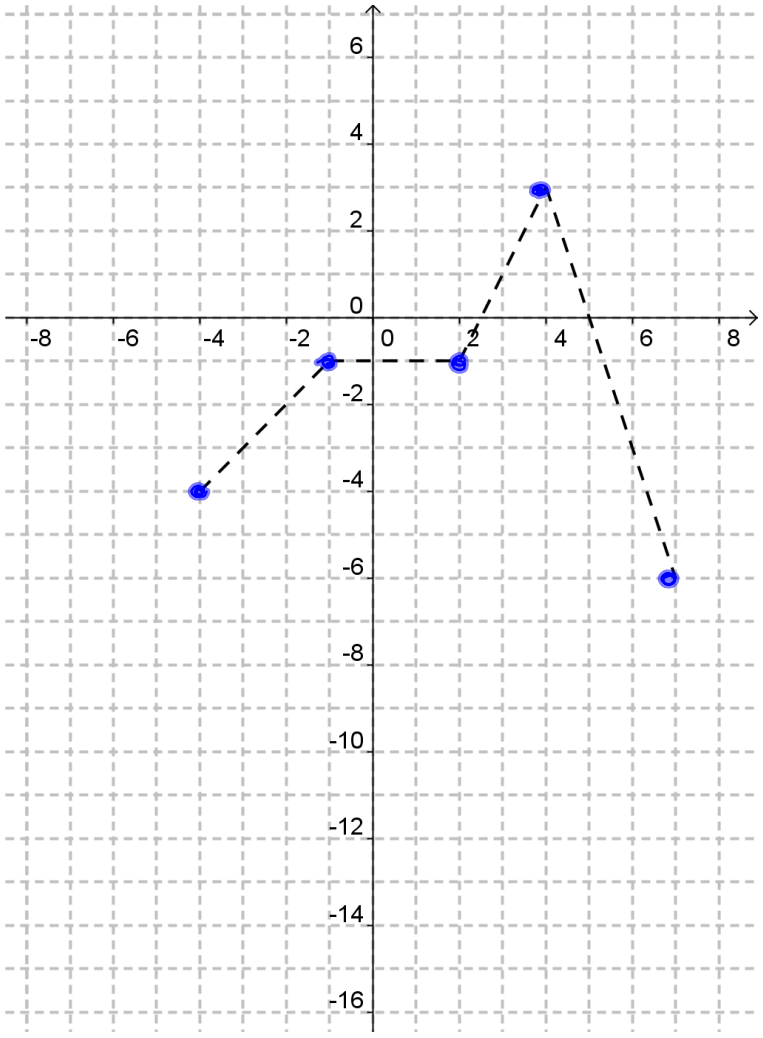
(-4, -1)	(-2, 0)	(2, 8)	(4, 8)
↓ v. reflect	↓	↓	↓
(-4, 1)	(-2, 0)	(2, -8)	(4, -8)
↓ left 1	↓	↓	↓
(-5, 1)	(-3, 0)	(1, -8)	(3, -8)
↓ down 2	↓	↓	↓
(-5, -1)	(-3, -2)	(1, -10)	(3, -10)

Ex: The graph on the right shows the function $y = f(x)$
Given $y = 2f(x) - 1$, describe the transformations and apply them to the function.

(Use a table of values with the key points, if you want to!)

① V. stretch x 2

② V. Shift down 1



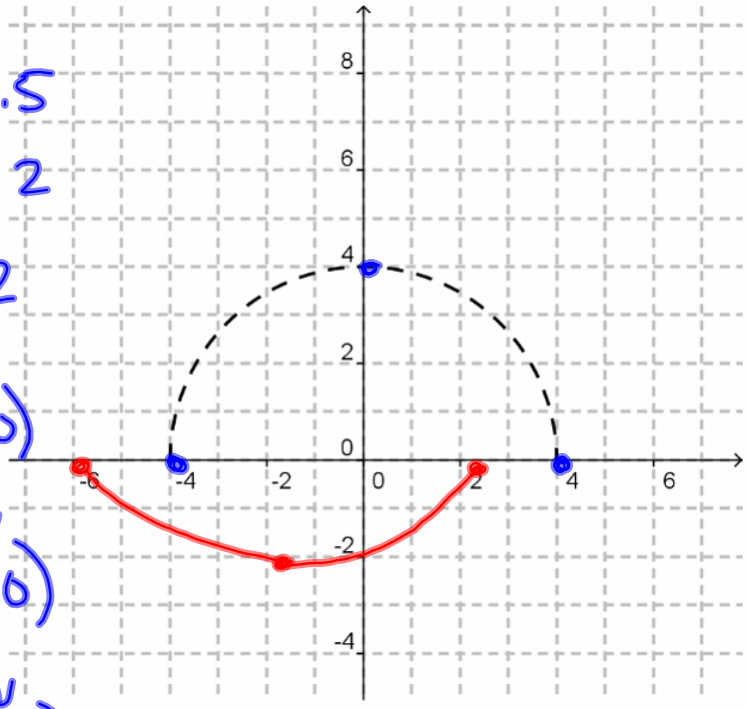
Ex: The graph on the right shows the function $f(x)=\sqrt{16-x^2}$.
Given $y=-0.5f(x+2)$, describe the transformations and apply them to the function.

(Use a table of values with the key points, if you want to!)

① v. reflect

② v. scaling by 0.5
or compression by 2

③ h. shift left 2



(-4,0)	(0,4)	(4,0)
↓	↓	↓
(-4,0)	(0,-4)	(4,0)
↓	↓	↓
(-4,0)	(0,-2)	(4,0)
↓	↓	↓
(-6,0)	(-2,-2)	(2,0)

Assigned Work:

p.241 # 8b, 9b, 5abd, 4abce

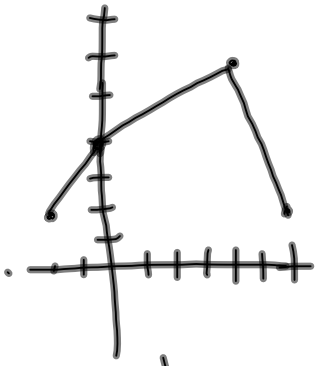
4a) $y = f(x-4)+2$

$(-6,0) \to (-2,0) \to (-2,2)$

$(-4,2) \to (0,2) \to (0,4)$

$(0,4) \to (4,4) \to (4,6)$

$(2,0) \to (6,0) \to (6,2)$



Right 4 Up 2

$D:\{x \in R \mid -2 \le x \le 6\}$

$R:\{y \in R \mid 2 \le y \le 6\}$

8b) $f(x) = x^2$

$2f(x+5)-4$

function notation

↳ stretch (v) x 2

↳ left 5

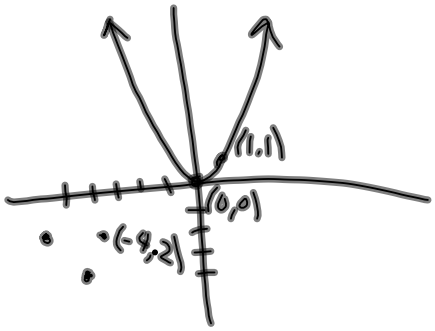
↳ down 4

$(1,1)$

$(1,2) \times 2^y$

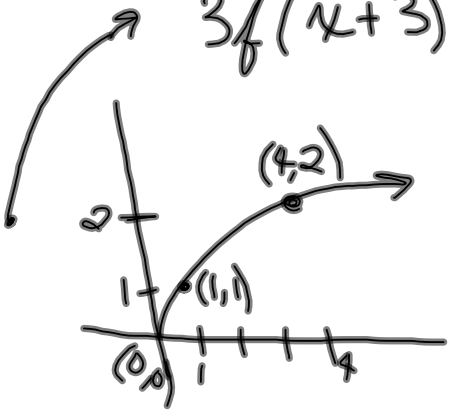
$(-4,2) - 5^x \quad (-4,-2) - 4^y$

$y = 2(x+5)^2 - 4$



9b) $f(x) = \sqrt{x}$

$3f(x+3)+2$



$(0,0) \to (0,0) \to (-3,0) \to (-3,2)$

$(1,1) \to (1,3) \to (-2,3) \to (-2,5)$

$(4,2) \to (4,6) \to (1,6) \to (1,8)$