Inverse of a Function

March 1/2012

Recall: A relation is a set of ordered pairs.

The inverse of a relation can be found by interchanging the domain and range of the relation.

In other words, swap the x- and y-values for each point.

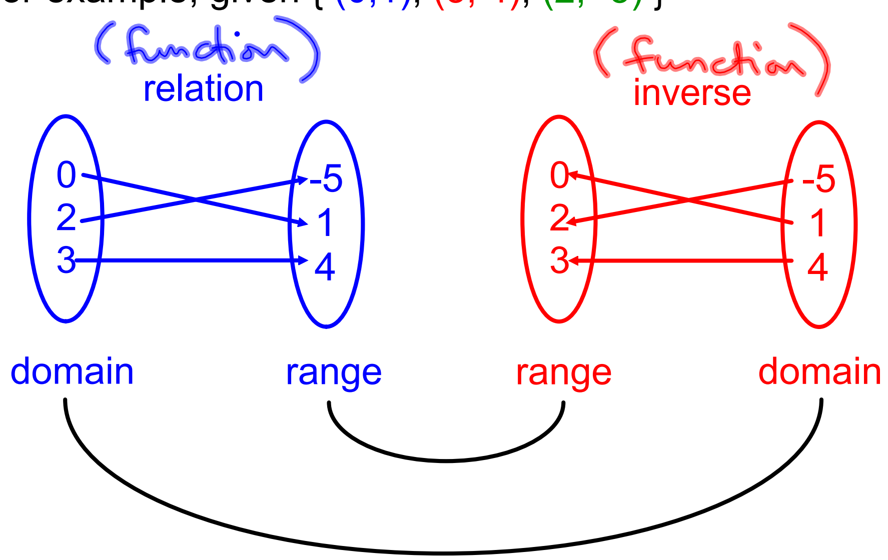
Ex.1 Determine the inverse of { (0,1), (3, 4), (2, -5) }.

The inverse is { (1, 0), (4, 3), (-5, 2) }.

We can represent the relation, and its inverse, using an arrow diagram or mapping diagram to show the relationship between domain and range.

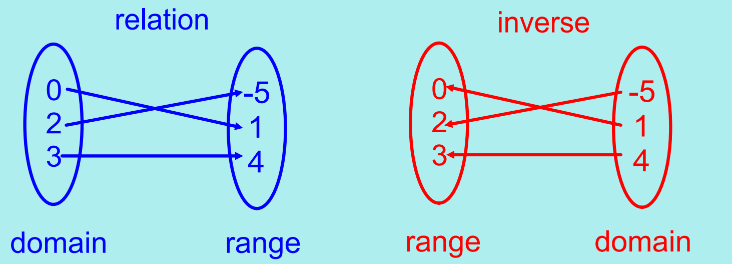
To find the inverse, reverse the direction of the lines.

For example, given { (0,1), (3, 4), (2, -5) }



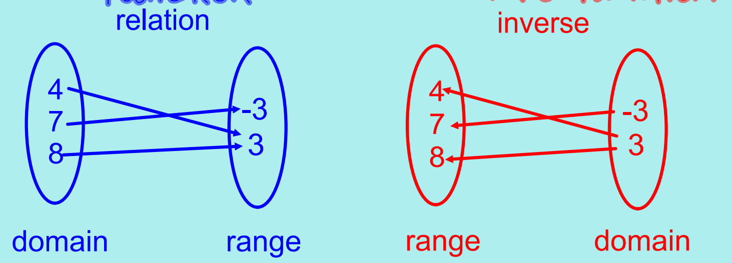
A mapping diagram can be used to determine if a relation is a function.

If there is only one arrow from each item in the domain, then it is a function.



relation is a function — inverse is a function

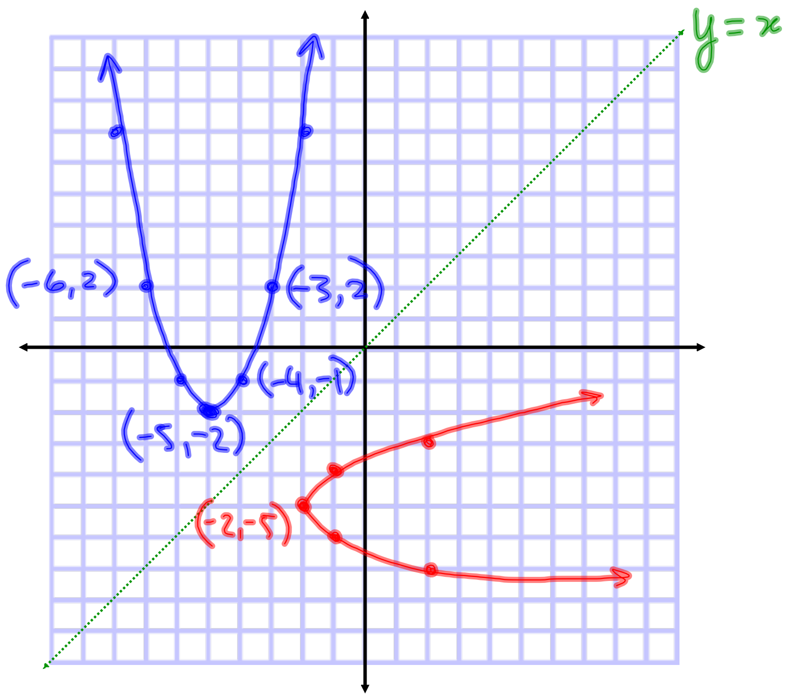
function — not a function



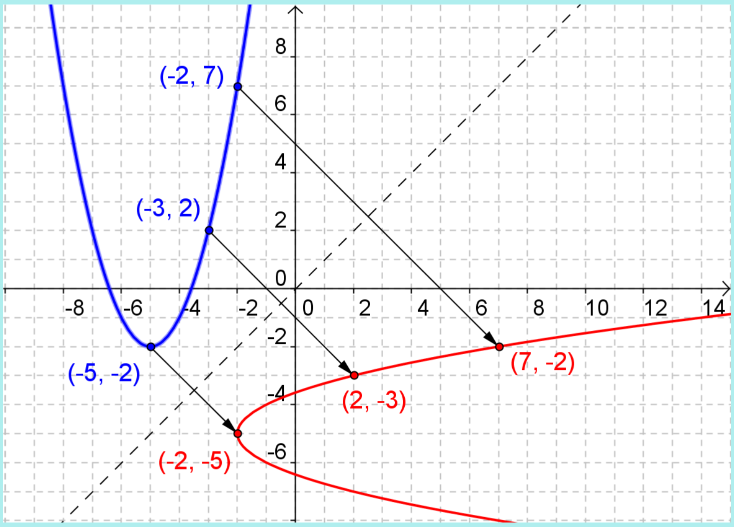
relation is a function — inverse is NOT a function

Graphically, swapping the x- and y-values is equivalent to reflecting the relation across the line y = x.

Ex.2 Find the inverse of $y = (x+5)^2 - 2$



Find the inverse of $y=(x+5)^2-2$ graphically



Notice that the original points and the reflected (swapped) points are equidistant (equal distance) to the line y = x.

The inverse (red) fails the vertical line test, and is not a function.

Recall: A function is a special type of relation where each element in the domain corresponds to a single value in the range.

For an inverse function, each value in the range corresponds to a single value in the domain.

If the inverse of the function, $f(x)$, is also a function, it is given the special designation of inverse function, $f^{-1}(x)$.

Note: In the inverse notation, the "-1" is not an exponent!

For example:

$$x^{-1}=\frac{1}{x} \qquad f^{-1}(x)\neq\frac{1}{f(x)}$$

Algebraically, swap the x- and y-variables, then rearrange the new equation for y.

If the original equation is in function notation, first change the function to y.

Ex.3 Find the inverse of $f(x) = 4x + 3$

$y = 4x + 3$

Swap x, y

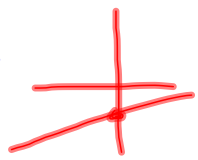
$x = 4y + 3$

$\frac{x-3}{4} = \frac{4y}{4}$

$y = \frac{x-3}{4}$

$y = \frac{x}{4} - \frac{3}{4}$

$y = \frac{1}{4}x - \frac{3}{4}$



$f^{-1}(x) = \frac{1}{4}x - \frac{3}{4}$

Ex.4. Find the inverse of

$f(x) = 3x^2 - 6$

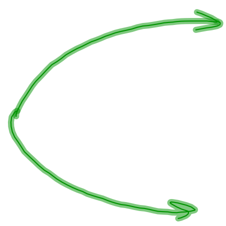
$y = 3x^2 - 6$

Swap x, y

$x = 3y^2 - 6$

$\frac{x+6}{3} = \frac{3y^2}{3}$

$y^2 = \frac{x+6}{3}$



$y = \pm\sqrt{\frac{x+6}{3}}$

$y = \sqrt{\frac{x+6}{3}}$

$y = -\sqrt{\frac{x+6}{3}}$

each x-value has 2 y-values

$\therefore$ not a function

A function and its inverse undo each other. From the previous example,

$$f(x) = 4x + 3 \qquad f^{-1}(x) = \frac{x-3}{4}$$

Ex. ~~4~~ 5 For each value, determine $y = f(x)$, then sub the y-value into $f^{-1}(x)$.

(a) x = 5 (b) x = -3 (c) x = 2.5

$f(5) = 4(5) + 3$
$= 23$

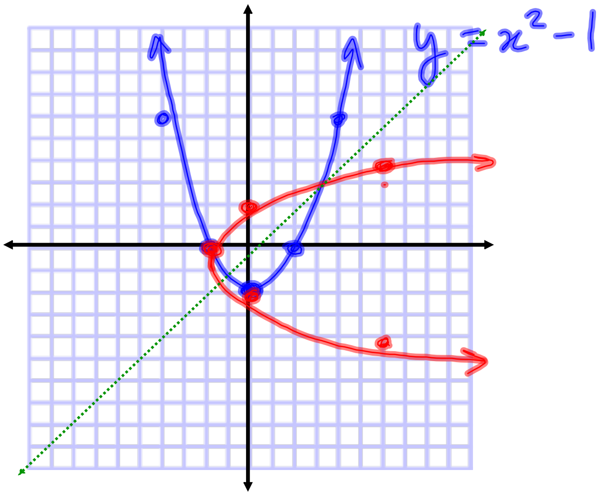
$f^{-1}(23) = \frac{23-3}{4}$
$= \frac{20}{4}$
$= 5$

Assigned Work:

p.215 # 1 - 3, 5odd, 11, 13odd
g g e a

p.215

3g.



$V(0,-1) \rightarrow (-1,0)$

$(1,0) \rightarrow (0,1)$

$(-1,0) \rightarrow (0,-1)$

5(g) $g(x) = \frac{5}{2}x - 4$