Determining Transformed Functions from Graphs

Recall: Given $y = a\,f\left[k\left(x-p\right)\right]+q$

1. vertical scaling by a for $a \neq 1$
   (includes vertical reflection for a<0)

2. horizontal scaling by $\frac{1}{k}$ for $k \neq 1$
   (incl. reflection)

3. horizontal translation by p

4. vertical translation by q

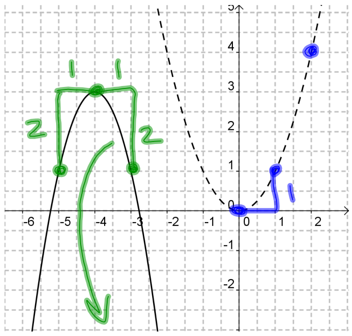
$$y = \underset{1}{a}\,f\left[\underset{2}{k}\left(x-\underset{3}{p}\right)\right]+\underset{4}{q}$$

For any single point, the transformations can be summarized as:

$$\left(x,y\right) \rightarrow \left(\underset{2}{\frac{x}{k}} + \underset{3}{p},\ \underset{1}{ay} + \underset{4}{q}\right)$$

Use logic, deductive reasoning, and linear systems of equations to determine values for a, k, p, and q.

Ex.1 Determine the transformation shown and express in function notation.



v.shift up 3

$q = 3$

y.reflect

$a < 0$

h.shift left 4

$p = -4$

$a = -2$

OR

$y = a(x+4)^2 + 3$

sub $(-3, 1)$

$1 = a(-3+4)^2 + 3$

$1 = a(1)^2 + 3$

$\boxed{-2 = a}$

Tips for parabolas: $y = a(x-p)^2 + q$

1. The vertex of the parent function is at (0, 0). The value zero is not affected by scaling (a or k), only translations (p or q). The vertex will be at (p, q).

2. Parabolas can ignore the horizontal scaling, k, because there is an equivalent 'a' value.

k

$y = (3x)^2$

$y = 9x^2$

a

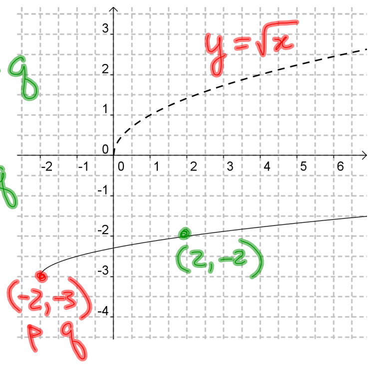
3. Use the step pattern (1, 3, 5, ...) from the vertex to determine the vertical scaling, 'a'.

Ex.2 Determine the transformations shown and express in function notation.

$y=af[k(x-p)]+q$

$\downarrow$

$y=a\sqrt{k(x-p)}+q$



$y=a\sqrt{x+2}-3$	$y=\sqrt{k(x+2)}-3$
sub(2,-2)	sub(2,-2)
$-2=a\sqrt{2+2}-3$	$-2=\sqrt{k(2+2)}-3$
$1=a(2)$	$1=\sqrt{4k}$
$a=\frac{1}{2}$	$1=4k$ $1=2\sqrt{k}$
	$k=\frac{1}{4}$ $\frac{1}{2}=\sqrt{k}$
	$\frac{1}{4}=k$
$y=\frac{1}{2}\sqrt{x+2}-3$	$y=\sqrt{\frac{1}{4}(x+2)}-3$

$\sqrt{k}$

a^2

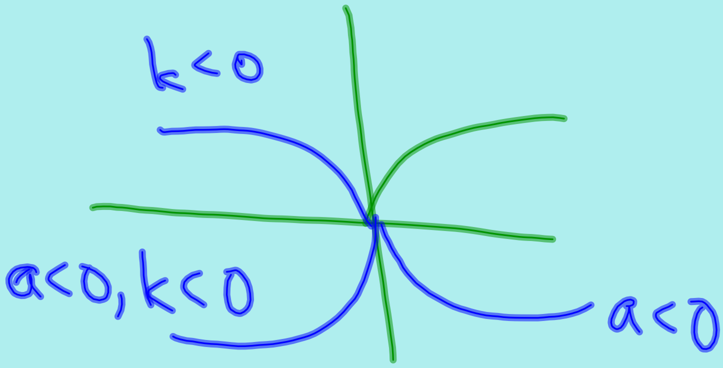
Tips for radicals: $y=a\sqrt{k(x-p)}+q$

1. The parent function starts at (0, 0), just like a parabola. The value zero is not affected by scaling (a or k), only translations (p or q).

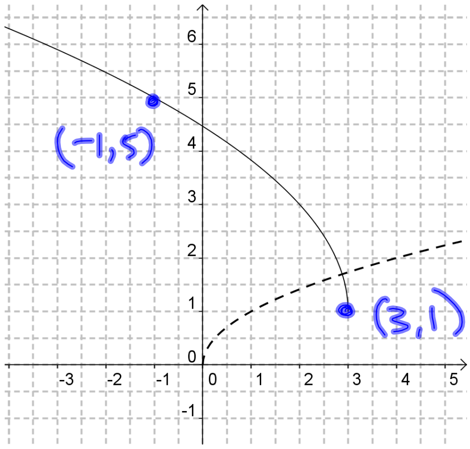
2. The sign of 'a' and 'k' are <u>both important</u> for reflections.

3. $a=1$ or $a=-1$

   Use ~~one of 'a' or~~ 'k' for scaling. The horizontal scaling is more likely to give a "nice" (integer) value.



Ex.3 Determine the transformations shown and express in function notation.



$p=3 \quad q=1$

$a>0 \quad k<0$

set $a=1$

$y=\sqrt{k(x-3)}+1$

Sub $(-1,5)$

$5=\sqrt{k(-1-3)}+1$

$4=\sqrt{-4k}$

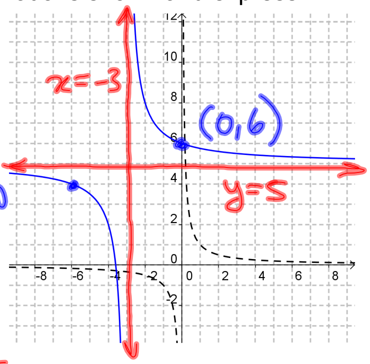
$16=-4k$

$\frac{16}{-4} = \frac{-4k}{-4}$

$k=-4$

$y=\sqrt{-4(x-3)}+1$

Ex.4 Determine the transformations shown and express in function notation.



$f(x)=\frac{1}{x}$

$y=af[k(x-p)]+q$

$\downarrow$

$y=\frac{a}{k(x-p)}+q$

$k=1 \quad p=-3 \quad q=5$

$y=\frac{a}{x+3}+5$

Sub$(0,6)$

$6=\frac{a}{0+3}+5$

$1=\frac{a}{3}$

$\boxed{a=3}$

$y=\frac{3}{x+3}+5$

Tips for rationals: $y = \dfrac{a}{k\left(x-p\right)} + q$

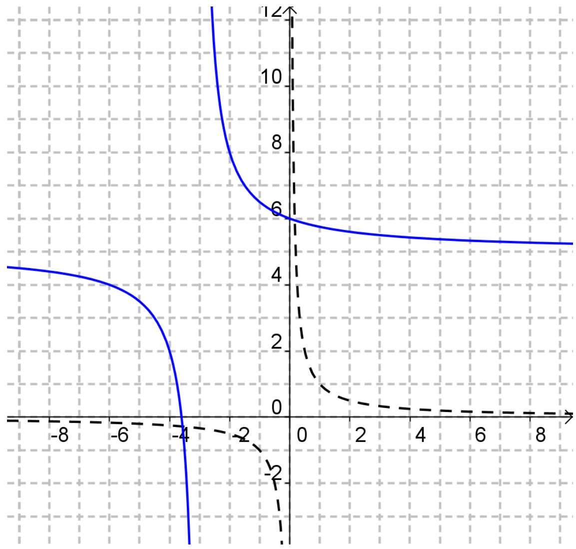
1. The parent function has asymptotes at x=0 and y=0.
   The new asymptotes will be at x = p and y = q.

2. Use only <u>one of</u> 'a' or 'k' for scaling and reflection.

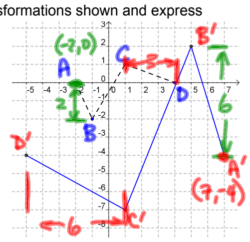
$$y = \frac{1}{3\left(x-2\right)} \Leftrightarrow y = \frac{1}{3} \times \frac{1}{x-2}$$

$k = 3$ $\qquad\qquad$ $a = \dfrac{1}{3}$

Ex.5 Determine the transformations shown and express
in function notation.



Ex.6 Determine the transformations shown and express in function notation.



$a<0 \quad k<0$

v. scaling by 3

$a=-3$

h. scaling ×2

$k=-\frac{1}{2}$

h. scaling by $\frac{1}{k}$

$$(x,y)\rightarrow\left(\frac{x}{k}+p,\ ay+q\right)$$

$x \rightarrow \frac{x}{k}+p \qquad y \rightarrow ay+q$

$x \rightarrow \frac{x}{-\frac{1}{2}}+p \qquad y \rightarrow -3y+q$

$x \rightarrow -2x+p \qquad y \rightarrow -3y+q$

start x, final x

$-2 \rightarrow -2(-2)+p = 7$ x-coord of A'

x-coord of A

$4+p=7$

$p=3$

$0 \rightarrow -4$ y-coord of A'

$0 \rightarrow -3(0)+q=-4$

y-coord of A

$q=-4$

$a=-3 \quad k=-\frac{1}{2} \quad p=3 \quad q=-4$

$y=af[k(x-p)]+q$

$y=-3f[-\frac{1}{2}(x-3)]-4$

$$y=af[k(x-p)]+q$$

$$(x,y)\rightarrow(x,ay)$$

$$\rightarrow\left(\frac{x}{k},\ ay\right)$$

$$\rightarrow\left(\frac{x}{k}+p,\ ay\right)$$

$$\rightarrow\left(\frac{x}{k}+p,\ ay+q\right)$$