Problem Solving with Rational Functions

Ex.1. A consultant has issued an environmental report on the cost of cleaning up a property that was previously the site of a chemical factory. Costs can increase dramatically depending on the percent of pollutants that needs to be removed.

Her report gives the cost, *C*, in dollars, of removing *p*% of the pollutants from the site as:

$$C(p) = \frac{50000}{100 - p}$$

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a) What is the cost of removal for half of the pollutants? $C(p) = \frac{50000}{100 - p}$

b) What is the cost of removal for 90% of the pollutants?

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c) Would it be affordable to remove all of the pollutants? $C(p)=\frac{50000}{100-p}$

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Ex.2 Fred drove his car a distance of $2x$ km in 3 hours. Later, he drove a distance of (x + 100) km in 2 hours.

a) Write an expression for the first speed.

Use the equation:

$$\text{speed}=\frac{\text{distance}}{\text{time}}$$

b) Write an expression for the second speed.

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Ex.2 Fred drove his car a distance of $2x$ km in 3 hours. Later, he drove a distance of $(x + 100)$ km in 2 hours.

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

c) Write a simplified expression for the difference between the first speed and the second speed.

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Ex.2 Fred drove his car a distance of $2x$ km in 3 hours. Later, he drove a distance of $(x + 100)$ km in 2 hours.

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

d) Determine the value(s) of x for which the speed was greater for the second trip.

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Ex.3 A jet flies along a straight path from Toronto to Montreal and back again. The straight-line distance between these cities is 540 km. On Monday, the jet made the round trip when there was no wind. On Friday, it made the round trip when there was a constant wind blowing from Toronto to Montreal at 80 km/h. While travelling in still air, the jet travels at constant speed.

Which round trip takes less time?

$v = \frac{d}{t}$

$d = vt$

$t = \frac{d}{v}$

Monday:

$t_M = \frac{540}{v} + \frac{540}{v}$

$t_M = \frac{1080}{v}$

Friday:

$t_F = \frac{540}{v+80} + \frac{540}{v-80}$

$t_F = \frac{540(v-80)+540(v+80)}{(v+80)(v-80)}$

$t_F = \frac{540v - 43200 + 540v + 43200}{(v+80)(v-80)}$

$= \frac{1080v}{(v+80)(v-80)}$

*Compare t_M to t_F

$\frac{1080}{v} \times \frac{v}{v}$ $\qquad \frac{1080v}{v^2-6400}$

$= \frac{1080v}{v^2}$

$\frac{1}{a} \qquad \frac{1}{b}$

compare v^2 to $v^2 - 6400$

parabola

parabola shifted down
* smaller!

$\therefore t_F$ (smaller denominator) must be larger.

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$t_M = \frac{1080}{v} \qquad t_F = \frac{1080v}{v^2-6400}$

$t_M - t_F = \frac{1080}{v} - \frac{1080v}{v^2-6400}$

$= \frac{1080(v^2-6400) - 1080v(v)}{v(v^2-6400)}$

$= \frac{1080v^2 - 6912000 - 1080v^2}{v(v^2-6400)}$

$= \frac{-6912000}{v(v^2-6400)}$

$v \neq 0, 80, -80$
(pure math)

$v > 80$
(real world)

denominator always positive

$t_M - t_F$ always negative

$\therefore t_F$ is larger. (longer time)

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Assigned Work:

- problem on worksheet
- problems from text (also listed on WS)

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