Exploring Exponential Growth

Activity 1:

In this activity, you will investigate the number of different pizzas that can be created for a given number of available toppings.

INSTRUCTIONS

1. Fill in the chart below by drawing all the different pizzas that can be created by choosing all, some or none of the available toppings indicated.

Toppings Available	Different Pizza Combinations	
None	N	1
Cheese	N, C	2
Cheese, Pepperoni	N, C, P, CP	4
Cheese, Pepperoni, Mushrooms	N, C, P, CP, M, CM, PM, CPM	8
Cheese, Pepperoni, Mushrooms, Bacon		16

2. a) Use your information from question 1 to complete the table below

Number of Available Toppings	Number of Different Pizzas	First Differences $(y_2 - y_1)$	Ratio $\left(\frac{y_2}{y_1}\right)$
0	1		
1	2	2 - 1 = 1	$\frac{2}{1} = 2$
2	4	4 - 2 = 2	$\frac{4}{2} = 2$
3	8	8 - 4 = 4	$\frac{8}{4} = 2$
4	16	16 - 8 = 8	$\frac{16}{8} = 2$

b) What do the First Differences tell us about these data? Why?

not a linear relation because 1st diff are not the same.

c) Is there a constant ratio between consecutive values in the column titled Number of Different Pizzas?

yes, a ratio of 2.

d) How does this value relate to the pattern in the First Differences?

1st diff are doubling (x2), ratio is 2.

e) Predict how many different pizzas could be created from 5 toppings. Justify your answer.

5 toppings → 32 combinations

each topping doubles possibilities

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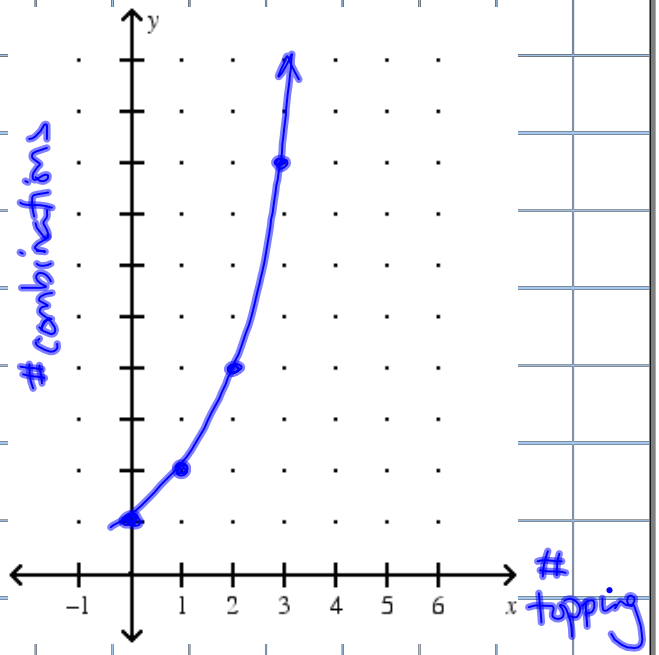
3. Neatly sketch a graph of your results from question 2 on a grid showing the first quadrant. Draw a smooth curve through the points.

(Note: These are DISCRETE data; however, the smooth curve assists in seeing the general shape of the graph.)



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4. Use the graph to describe the shape of the curve. Use words such as the following: increasing, decreasing, quickly, slowly.

increasing quickly

5. **Complete this statement**: Each time the number of toppings increases by 1, the number of different pizza combinations double

6. Predict how many different pizzas can be created if there are nine available toppings. Clearly explain how you made your prediction. 512

$2^9 = 512$

7. If a restaurant owner would like to offer 200 different pizza combinations, what is the minimum number of available toppings she would need? Explain your reasoning.

$2^8 = 256, 2^7 = 128$, ∴ 8 toppings

8. Your local pizza parlour offers you the choice of 15 different toppings. If you were to eat a different pizza every day, how many years would it take for you to try every possible one? (Hint: There are 365 days in a year.) Does this answer surprise you? Why or why not?

$2^{15} = 32768$ (days)

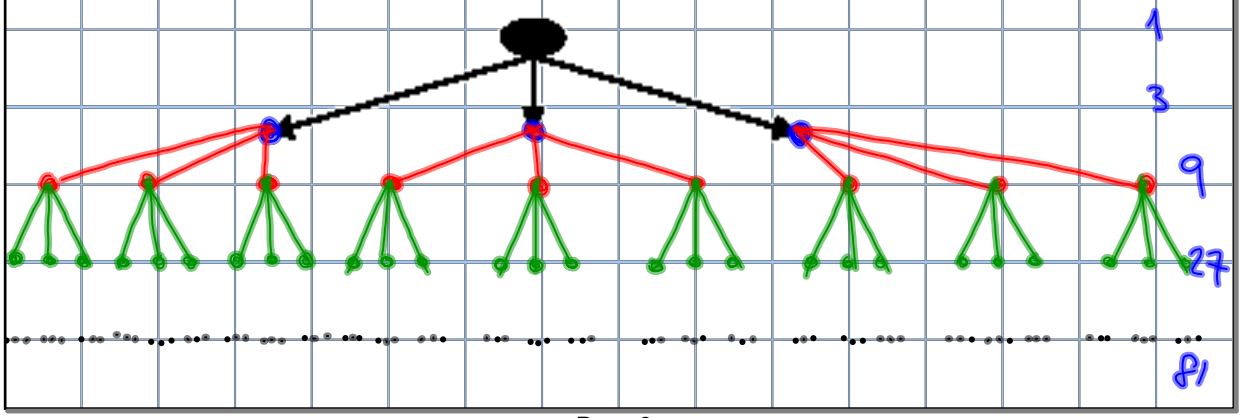
$\frac{32768}{365} = 89.8$ years.

Activity 2:

In this activity, you and your partner will use a Tree Diagram to simulate the effect of "telling three friends, who each tell three friends, who each tell three friends" and so on.

INSTRUCTIONS

1. A letter is e-mailed out to three friends. Each of these three recipients will then e-mail it to three new friends. Continue this pattern in order to complete the first four rounds in the Tree Diagram below. (Hint: Use tiny dots to represent each e-mail that is sent so that you will have enough space to draw out the entire Tree Diagram or write the number at each branch.)



2. a) Use your information from question 1 to complete the table below

Round	Number of Letters e-mailed	First Differences	Ratio
1	3		
2	9	9-3=6	$\frac{9}{3}=3$
3	27	27-9=18	$\frac{27}{9}=3$
4	81	81-27=54	3
5	243	243-81=162	3

b) Consider the Tree Diagram and the data in the first four rows of the table. Predict the Number of Letters e-mailed in Round 5. Justify your prediction.

243

c) Comment on the patterns you see in the Number of Letters e-mailed column and the First Differences column.

tripling each round

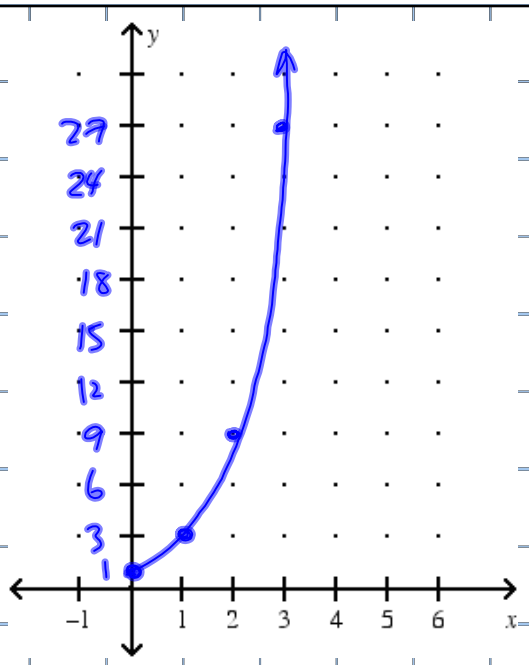
d) Is there a constant ratio between consecutive values in the Number of Letters e-mailed column?

ratio is 3

e) Comment on the value of this constant ratio.

3. Neatly sketch a graph of your results from question 2 on a grid showing the first quadrant. Draw a smooth curve through the points.

(Note: These are DISCRETE data; however, the smooth curve assists in seeing the general shape of the graph.)



4. **Complete this statement**: As the number of rounds increases by 1, the number of letters e-mailed triples

5. Predict how many letters will be e-mailed during the 9th round. Show how you determined this.

$3^9 = 19683$

6. During which round will the number of letters e-mailed exceed 200 000 for the first time? Show how you determined this.

$3^{11} = 177147$

$3^{12} = 531441$

$\therefore$ exceed 200000 after 12 rounds.

7. Pyramid schemes work similarly to this e-mail simulation in that individuals must find others willing to invest in (or purchase a product from) a "company".

Consider a pyramid scheme where people are asked to invest $1000 each and are required to find four more investors to do the same. How much **total** money will be invested in this "company" after three rounds? (Hint: Draw a tree diagram.)

NOTE: Pyramid schemes are illegal because they usually involve fraud.

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Homework:

Activity #4, 5*

Apr 3-2:59 PM