Unit 4 - Exponential Functions

Review of Exponent Laws

March 30/ 2012

Exercises: p.9 #1-9(odd), 12, 16

Recall:
A **power** is a product of identical factors and consists of two parts: a **base** and an **exponent**.

base $\rightarrow 2^3 \leftarrow$ exponent

base = the identical factor
exponent = how many factors there are altogether

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Ex.1 Evaluate.

a) $(-3)^3$
$=(-3)(-3)(-3)$
$=-27$

b) $3^3 = 27$

c) -2^4
$=(-1)(2^4)$
$=-16$

d) $(-2)^4$
$=(-2)(-2)(-2)(-2)$
$=16$

e) 3^4
$=81$

f) $\left(\frac{4}{3}\right)^2 = \left(\frac{4}{3}\right)\left(\frac{4}{3}\right)$
$=\frac{16}{9}$

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Rule #1: Multiplication of Powers with the same base

To investigate the rule let us look at a specific example and go through the process of expanding before simplifying.

$$(3^1)(3^2) = (3)[(3)(3)]$$
$$= 3^3$$

The Rule: $(a^x)(a^y) = a^{x+y}$

In words: when multiplying powers with the same base, add exponents.

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Rule #2: Division of Powers with the same base

To investigate the rule let us look at a specific example and go through the process of expanding before simplifying.

$$\cancel{3^1 \div 3^2} \quad 3^2 \div 3^1 = \frac{(3)\cancel{(3)}}{\cancel{3}}$$
$$= 3^1$$

The Rule: $a^x \div a^y = \dfrac{a^x}{a^y}$

$$= a^{x-y},\ a \neq 0$$

In words: when dividing powers with the same base, subtract exponents.

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Rule #3: Power of a Power

To investigate the rule let us look at a specific example and go through the process of expanding before simplifying.

$$(3^2)^4 = (3^2)(3^2)(3^2)(3^2)$$
$$= 3^8$$

The Rule: $(a^x)^y = a^{xy}$

In words: when having a power to an exponent, multiply the exponents.

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Ex.2 Simplify. Express your final answer with positive exponents.

a) $(4^{-6})(4^4)$

$= 4^{-6+4}$

$= 4^{-2}$

b) $\dfrac{(-3)^2}{(-3)^{-3}}$ $= (-3)^{2-(-3)}$

$= (-3)^5$

c) $(5^{-2} \times 5^4)^{-2}$

$= (5^2)^{-2}$

$= 5^{-4}$

d) $(x^{-3}\, y^5)^{-3}$

$= x^9 y^{-15}$

e) $(3a^2b^1)(-2a^3b^4)$

$= -6a^5b^5$

~~f) $(-a^5b)^{-2}(-ab^{-2})^2$~~

=

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Patterns in Powers of 2

exponent

-5 -4 -3 -2 -1 0 1 2 3 4 5

2^0 2^1 2^2 2^3

1 2 4 8

×2 ×2 ×2

÷2 ÷2 ÷2

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Patterns in Powers of 2

exponent

-5 -4 -3 -2 -1 0 1 2 3 4 5

2^0 2^1 2^2 2^3

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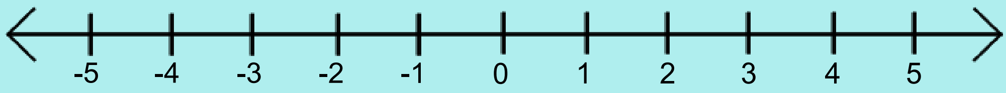
Patterns in Powers of 2

exponent



2^{-3} 2^{-2} 2^{-1} 2^0 2^1 2^2 2^3

$\frac{1}{8}$ $\frac{1}{4}$ $\frac{1}{2}$ 1 2 4 8

$\div 2$

$\frac{1}{2^3}$ $\frac{1}{2^2}$ $\frac{1}{2^1}$

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Rule #4: Identity Rule

What exponent does not change the value of a power?

The Rule: $a^1 = a$

In words: anything to the exponent of 1 is equal to itself.

Rule #5: Zero Exponent

Lets look at the expanded form of powers and find a pattern:

$2^3 =$ 8

$2^2 =$ 4 $\quad\div 2$

$2^1 =$ 2

then $2^0 =$ 1

The Rule: $a^0 = 1,\ a \neq 0$ $\quad 0^0$ is undefined

In words: anything to the exponent of zero is 1.
This is because an exponent of zero means you are dividing the base by itself.

Rule #6: Negative Exponent

Now continue the pattern from the previous rule to determine the effect of a negative in the exponent:

$2^0 =$ 1

$2^{-1} =$ $\frac{1}{2}$

$2^{-2} =$ $\frac{1}{4}$

The Rule: $a^{-x} = \left(\frac{1}{a}\right)^x,\ a \neq 0$

In words: a negative exponent requires you to find the reciprocal of the base.

Rule #7: Distributive Rule (for powers with different bases)

To investigate the rule let us look at a specific example and go through the process of expanding before simplifying.

$$\left(7^2 \cdot 2^5\right)^3 = \left(7^2 \cdot 2^5\right)\left(7^2 \cdot 2^5\right)\left(7^2 \cdot 2^5\right)$$
$$= 7^6 \cdot 2^{15}$$

$$\left(\frac{7^2}{2^5}\right)^3 =$$

The Rules:

(a) $(ab)^x = (a^x)(b^x)$

(b) $\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x},\ b \neq 0$

(c) $\left(a^m b^n\right)^p =$

(d) $\left(\frac{a^m}{b^n}\right)^p =$

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The exponent laws also work if you have polynomials instead of numbers as exponents.

Ex.3 Simplify

a) $(x^3)^{2a+4}$

$= x^{3(2a+4)}$

$= x^{6a+12}$

b) $(x^{a+5})(x^{3a+1})$

$= x^{(a+5)+(3a+1)}$

$= x^{4a+6}$

c) $(x^{4m-3n}) \div (x^{m+5n})$

$= x^{(4m-3n)-(m+5n)}$

$= x^{3m-8n}$

d) $x^y(x^{y+1})^{y+2}(1/x)^{6y}$

$= x^y\, x^{(y+1)(y+2)}\, x^{-6y}$

$= x^{y+(y+1)(y+2)-6y}$

$= x^{-5y+(y^2+3y+2)}$

$= x^{y^2-2y+2}$

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The exponent laws:
(for powers with the same base)

$$(a^x)(a^y) = a^{x+y}$$

$$a^x \div a^y = \frac{a^x}{a^y} = a^{x-y},\ a \neq 0$$

$$a^{-x} = \frac{1}{a^x},\ a \neq 0$$

$$\left(a^x\right)^y = a^{xy}$$

$$a^0 = 1,\ a \neq 0$$

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The exponent laws:
(for powers with different bases)

$$(ab)^x = \left(a^x\right)\left(b^x\right)$$

$$\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x},\ b \neq 0$$

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Assigned Work:

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p.9 # 9(b)

$$\left(\frac{m^{-3}}{n}\right)^0 = 1$$

(d) $$\frac{3^{-3}+3^{-4}}{3^{-5}} = \frac{\frac{1}{3^3}\times\frac{3}{3} + \frac{1}{3^4}}{\frac{1}{3^5}}$$

$$= \frac{\frac{3}{3^4}+\frac{1}{3^4}}{\frac{1}{3^5}}$$

$$= \frac{3+1}{3^4}\times\frac{3^5}{1}$$

$$= \frac{4\times 3^{5-4}}{1}$$

$$= 4\times 3$$

$$= 12$$

$$\frac{1}{3^3} = \frac{1}{27}$$

$$\frac{3}{3^4} = \frac{3}{81} = \frac{1}{27}$$

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