

Recall: Exponent Laws (same base)

$$(a^x)(a^y) = a^{x+y} \qquad a^x \div a^y = \frac{a^x}{a^y} = a^{x-y}, \quad a \neq 0$$

$$a^{-x} = \frac{1}{a^x}, \quad a \neq 0 \qquad (a^x)^y = a^{xy}$$

$$a^0 = 1, \quad a \neq 0$$

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Recall: Exponent Laws (different base)

$$(ab)^x = (a^x)(b^x)$$

$$\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}, \quad b \neq 0$$

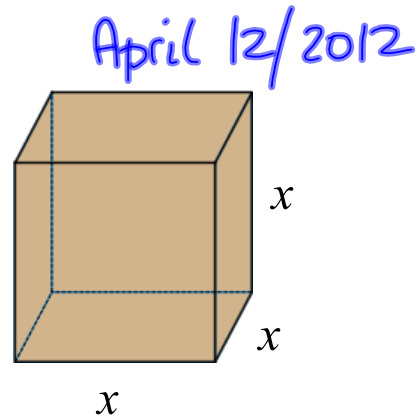
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Rational Exponents

For a cube of side length x ,

volume: $V(x) = x^3$

area of a side: $A_{side}(x) = x^2$



How could we represent the side length, x , as a power (i.e., exponential relation) of the volume or area?

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if $V = x^3$

then $x = V^n$

$V = x^3$

$V^1 = (V^n)^3$

$V^1 = V^{3n}$

$\frac{1}{3} = \frac{3n}{3}$

$n = \frac{1}{3}$

Since bases are the same, the exponents must be the same

$x = A^m$

$A = x^2$

$A = (A^m)^2$

$A^1 = A^{2m}$

$1 = 2m$

$m = \frac{1}{2}$

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Summary: A rational exponent is equivalent to a radical.

$$(b)^{\frac{1}{n}} = \sqrt[n]{b}, \text{ also called the } \underline{\text{nth root of } b},$$

where $n > 0$, n is an integer.

Note: Typical rules/restrictions apply, such as

- cannot divide by zero
- cannot take the square root of a negative

Ex.1 Express in radical notation, then evaluate.

(a) $10000^{\frac{1}{4}}$ (b) $(-8)^{\frac{1}{3}}$ (c) $49^{-\frac{1}{2}}$

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Ex.1 Express in radical notation, then evaluate.

(a) $10000^{\frac{1}{4}}$ (b) $(-8)^{\frac{1}{3}}$ (c) $49^{-\frac{1}{2}}$

$$= \sqrt[4]{10000}$$

$$= 10$$

$$= \sqrt[3]{-8}$$

$$= -2$$

$$\begin{aligned} &(-2)^3 \\ &= (-2)(-2)(-2) \\ &= -8 \end{aligned}$$

$$= \frac{1}{49^{\frac{1}{2}}}$$

$$= \frac{1}{\sqrt{49}}$$

$$= \frac{1}{7}$$

by convention,
do not write 2

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Ex.2 Express in radical notation, then evaluate.

$$\begin{aligned} \text{(a)} \quad 27^{\frac{2}{3}} &= \left(27^{\frac{1}{3}}\right)^2 \\ &= \left(\sqrt[3]{27}\right)^2 \\ &= (3)^2 \\ &= 9 \end{aligned}$$

$\frac{1}{3} \times \frac{2}{1} = \frac{2}{3}$

$= \sqrt[3]{27^2}$

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Ex.2 Express in radical notation, then evaluate.

$$\begin{aligned} \text{(b)} \quad (-27)^{\frac{4}{3}} &= \left[(-27)^{\frac{1}{3}}\right]^4 \\ &= \left(\sqrt[3]{-27}\right)^4 \\ &= (-3)^4 \\ &= 81 \end{aligned}$$

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Summary: The numerator of a rational exponent may be a value other than 1. Use exponent laws to express as a combination of a power and a radical.

$$(b)^{\frac{m}{n}} = (\sqrt[n]{b})^m = \sqrt[n]{b^m}$$

where n and m are integers, $n > 0$, $m > 0$

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Even & Odd Roots

Recall: It is not possible to take the square root of a negative number. Why?

No number multiplied by itself would be negative.

Similarly, you cannot take any even root (2, 4, 6, etc) of a negative number.

With odd roots (3, 5, 7, etc), it is possible to have positive or negative values under the root.

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Assigned Work:

handout # (1-6)(odd), 8, 10, 12, 14

2e, 6e 14a

3c

4c

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$$\begin{aligned} 2(e) \quad \sqrt[5]{-\frac{32}{243}} &= \left(-\frac{32}{243}\right)^{\frac{1}{5}} \\ &= \frac{(-32)^{\frac{1}{5}}}{(243)^{\frac{1}{5}}} \\ &= \frac{-2}{3} \end{aligned}$$

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$$3(c) \quad (-11)^2 (-11)^{3/4}$$

$$= (-11)^{2 + \frac{3}{4}}$$

$$= (-11)^{11/4}$$

DNE

$$2 + \frac{3}{4}$$

$$= \frac{8}{4} + \frac{3}{4}$$

$$= \frac{11}{4}$$

$$(-11^2)(-11^{3/4})$$

$$= + 11^{11/4}$$

$$-2^2 \quad (-2)^2$$

$$= -4 \quad = 4$$

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$$4(c) \quad \frac{\sqrt{28} \sqrt{4}}{\sqrt{7}} = \sqrt{\frac{28 \cdot 4}{7}}$$

$$= \sqrt{\frac{4 \cdot 4}{1}}$$

$$= \sqrt{16}$$

$$= 4$$

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$$\begin{aligned}
 \text{b(e)} \quad \frac{(16^{-2.5})^{-0.2}}{16^{3/4}} &= \frac{(16^{-5/2})^{-1/5}}{16^{3/4}} && \begin{array}{l} -\frac{5}{2} \times -\frac{1}{5} \\ = +\frac{1}{2} \end{array} \\
 &= \frac{16^{1/2}}{16^{3/4}} \\
 &= 16^{\frac{1}{2} - \frac{3}{4}} \\
 &= 16^{\frac{2}{4} - \frac{3}{4}} \\
 &= 16^{-\frac{1}{4}} \\
 &= \frac{1}{16^{1/4}} \\
 &= \frac{1}{\sqrt[4]{16}} \\
 &= \frac{1}{2}
 \end{aligned}$$

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$$\begin{aligned}
 \text{14(a)} \quad 9^{1/2} + 4^{1/2} &= (9+4)^{1/2} ? \\
 x^n + y^n &= (x+y)^n ? \\
 \text{LS} = 9^{1/2} + 4^{1/2} & \quad \text{RS} = (9+4)^{1/2} \\
 &= 3 + 2 &= (13)^{1/2} \\
 &= 5 &= \sqrt{13} \\
 \text{LS} &\neq \text{RS}
 \end{aligned}$$

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