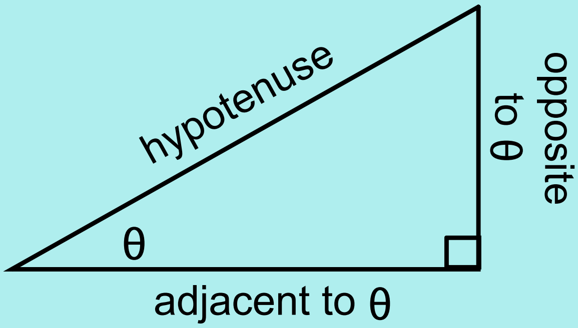
Recall:

For any angle of interest (θ), there are three (3) primary trigonometric ratios.

$$\text{sine of } \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\text{cosine of } \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

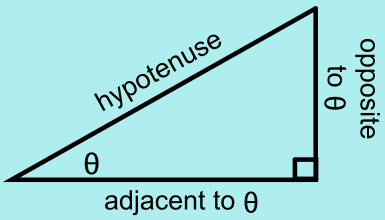
$$\text{tangent of } \theta = \frac{\text{opposite}}{\text{adjacent}}$$



S o h C a h T o a

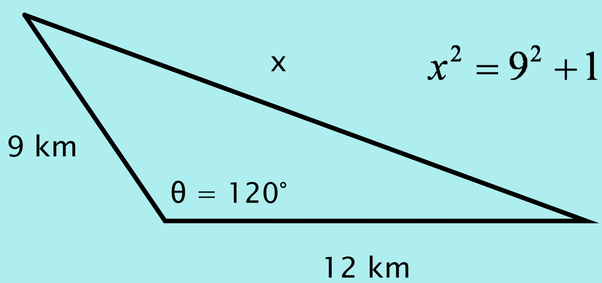
Apr 25-9:54 PM

Our reference triangle (as shown) is generally represented as an <u>acute triangle</u> (i.e., all angles $\leq 90°$).



But...

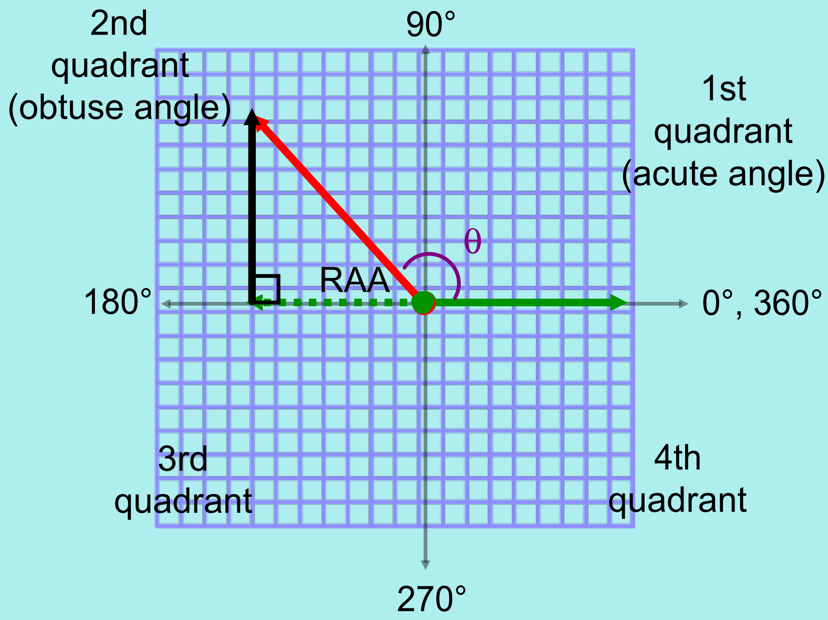
using the cosine law, we have solved triangles such as the one shown below... how?



$$x^2 = 9^2 + 12^2 - 2(9)(12)\cos(120°)$$

Apr 25-9:54 PM

To work with angles greater than 90°, we form a right-triangle using the <u>terminal arm</u> and the <u>related acute angle</u>.

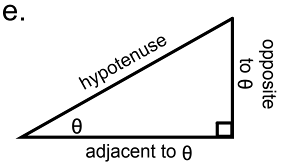


Apr 19-9:19 PM

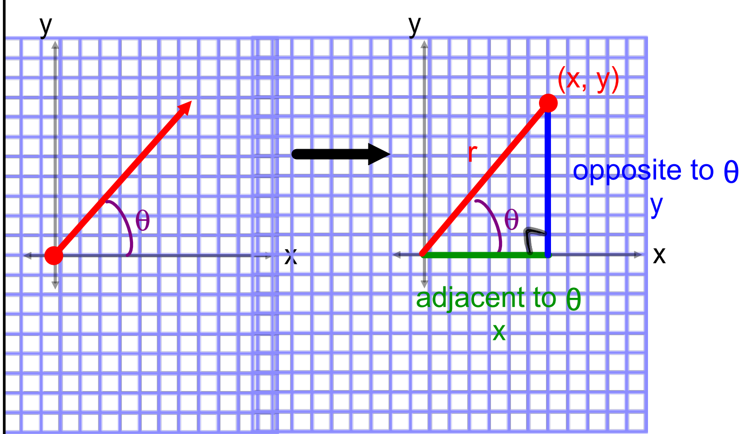
Trigonometry of Obtuse Angles

Apr. 30/2012

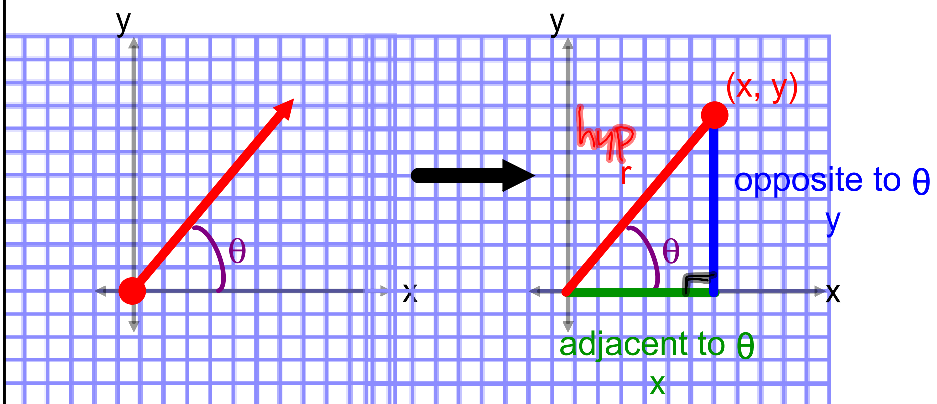
All trigonometric ratios are defined in terms of the sides of an acute right-triangle.



We can redefine the trig ratios for angles in standard position by drawing a right-triangle using the terminal arm.



Apr 19-9:13 PM



where: $r^2 = x^2 + y^2$

$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r} \qquad \tan\theta = \frac{y}{x}$$

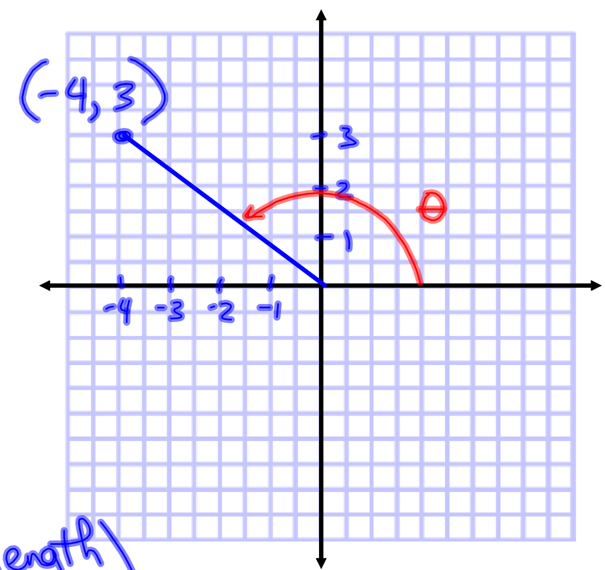
This definition refers only to coordinates (x, y) and the distance (r) from the origin to the point, and should be valid for any angle.

Apr 25-10:21 PM

Ex.1 The point (-4, 3) is on the terminal arm of an angle θ in standard position. Find sine and cosine for θ.

$\sin\theta = \frac{y}{r}$

$\cos\theta = \frac{x}{r}$



$r^2 = x^2 + y^2$

$r^2 = (-4)^2 + (3)^2$

$r^2 = 25$

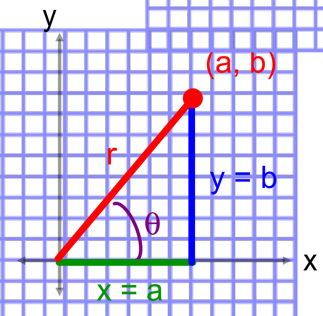
$r = 5$ (r is a length, $r > 0$)

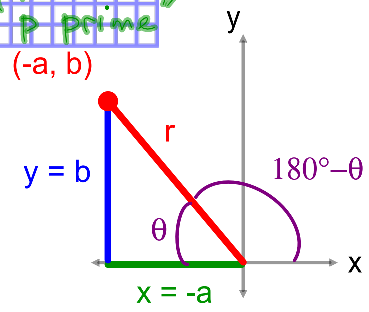
$\sin\theta = \frac{3}{5}$ $\cos\theta = \frac{-4}{5}$

Apr 25-10:48 PM

Focusing on obtuse angles ($90° < \theta < 180°$), consider the points P(a, b) and P'(-a, b).

"P prime"





How can we relate $\sin\theta$ to $\sin(180° - \theta)$?

$\sin\theta = \frac{y}{r}$
$= \frac{b}{r}$

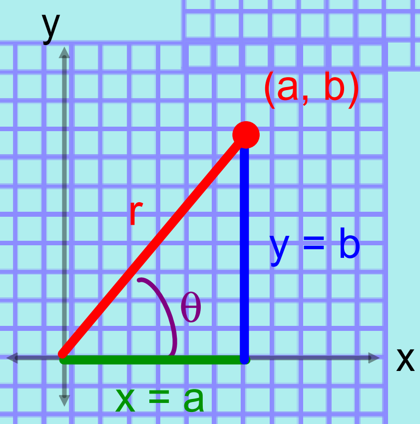
$\sin(180° - \theta) = \frac{y}{r}$
$= \frac{b}{r}$

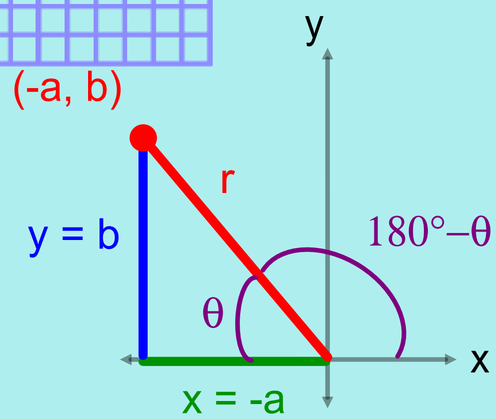
$\therefore \sin\theta = \sin(180° - \theta)$

* sine cannot distinguish between acute and obtuse angles.

Apr 25-10:27 PM

Focusing on obtuse angles ($90° < \theta < 180°$), consider the points P(a, b) and P'(-a, b).





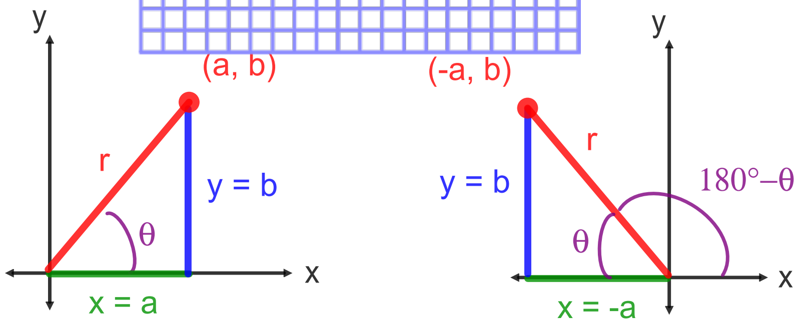
$$\sin\theta = \frac{b}{r}$$

$$\sin(180° - \theta) = \frac{b}{r}$$

$$\therefore \sin\theta = \sin(180° - \theta)$$

Apr 25-10:27 PM

Focusing on obtuse angles ($90° < \theta < 180°$), consider the points P(a, b) and P'(-a, b).



How can we relate $\cos\theta$ to $\cos(180° - \theta)$?

$\cos\theta = \frac{x}{r}$
$= \frac{a}{r}$

$\cos(180° - \theta) = \frac{x}{r}$
$= \frac{-a}{r}$

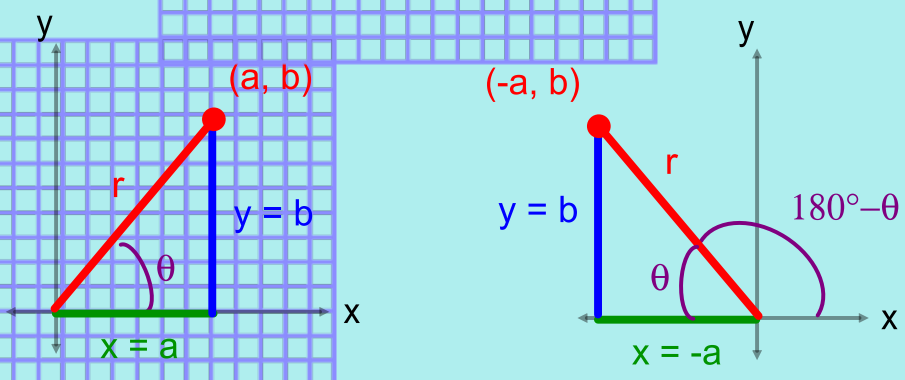
$\therefore \cos\theta = -\cos(180° - \theta)$

or

$-\cos\theta = \cos(180° - \theta)$

Apr 25-10:27 PM

Focusing on obtuse angles ($90° < \theta < 180°$), consider the points P(a, b) and P'(-a, b).



$\cos\theta = \frac{a}{r}$

$\cos(180° - \theta) = \frac{-a}{r}$

$\therefore \cos\theta = -\cos(180° - \theta)$

or

$-\cos\theta = \cos(180° - \theta)$

Apr 25-10:27 PM

Ex.2 Express each of the following as the trig ratio of an acute angle, then confirm your answer.

(a) sin(125°)

$\sin\theta = \sin(180° - \theta)$

$\sin 125° = \sin(180° - 125°)$

$= \sin(55°)$

$LS \doteq 0.8192$

$RS \doteq 0.8192$

$LS = RS$ ✓

(b) cos(160°)

$\cos\theta = -\cos(180° - \theta)$

$\cos 160° = -\cos(20°)$

$LS \doteq -0.9397$

$RS \doteq -(0.9397)$

$LS = RS$ ✓

Apr 25-11:14 PM

Ex.3 Find θ, if 0 ≤ θ ≤ 180°.

(a) sin θ = 0.25

$\theta = \sin^{-1}(0.25)$

$\theta \doteq 14.5°$

$\therefore \sin(14.5°) \doteq 0.25$

also

$\sin(180° - 14.5°) \doteq 0.25$

$\sin(165.5°) \doteq 0.25$

$\therefore \theta \doteq 14.5°$ or $\theta \doteq 165.5°$

(b) cos θ = -0.87

$\theta = \cos^{-1}(-0.87)$

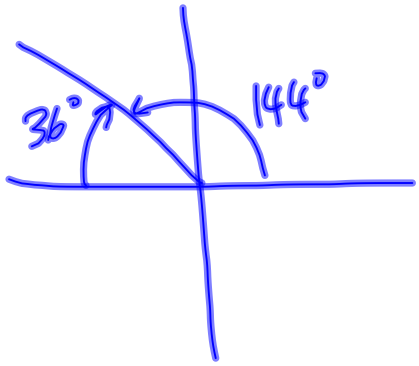
$\theta \doteq 150.5°$

Apr 25-11:14 PM

Assigned Work:

p.281 # 1
2odd (express in terms of RAA first)
3odd, 5, 6, 9, 12*

5, 6b, 9, 12

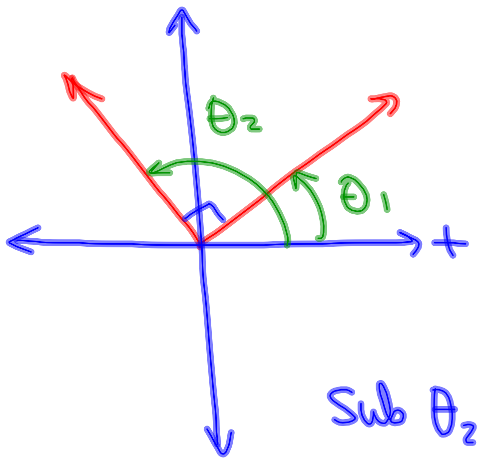


$\cos 144°$
$= -\cos(180° - 144°)$
$= -\cos(36°)$

$\cos\theta = -\cos(180° - \theta)$

Apr 21-12:17 AM

5. Supplementary angles → add to 180°
Complimentary angles → add to 90°



$\theta_1 + \theta_2 = 180°$ ①

$\theta_2 - \theta_1 = 90°$ ②

$2\theta_2 = 270°$

$\boxed{\theta_2 = 135°}$

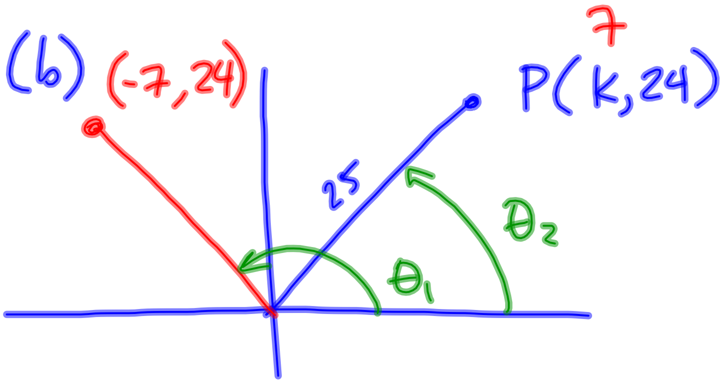
Sub $\theta_2 = 135°$ into ①

$\theta_1 + 135° = 180°$

$\boxed{\theta_1 = 45°}$

May 1-1:55 PM

6(b)



$$x^2+y^2=r^2$$
$$k^2+24^2=25^2$$
$$k^2=625-576$$
$$k^2=49$$
$$k=\pm 7$$

(b) $\cos\theta = \frac{x}{r}$

$$\cos\theta_1 = \frac{-7}{25}$$

$$\cos\theta_2 = \frac{7}{25}$$

May 1-2:01 PM

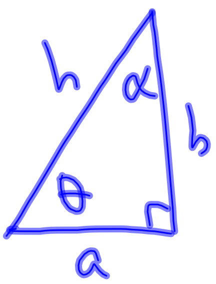
9.

$\sin 60^\circ \doteq 0.8660$ $\qquad \therefore \sin 60^\circ = \cos 30^\circ$

$\cos^{-1}(0.8660) \doteq 30^\circ$

$\sin 45^\circ \doteq 0.707$ $\qquad \therefore \sin 45^\circ = \cos 45^\circ$

$\cos^{-1}(0.707) \doteq 45^\circ$



$\sin 30^\circ = \cos 60^\circ$

$\sin 15^\circ = \cos 75^\circ$

$\sin\theta = \frac{b}{h}$ $\qquad \cos\alpha = \frac{b}{h}$

May 1-2:06 PM

12.

May 1-2:12 PM