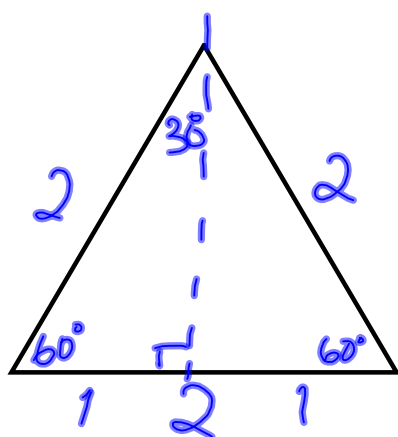# Special Triangles & Exact Values
## for Trig Ratios

Some angles occur so frequently that their trig ratios
should be known <u>exactly</u> (i.e., no decimals, no rounding).

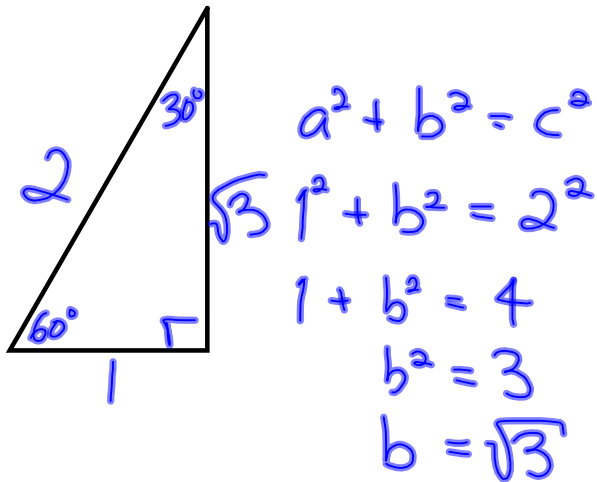The angles are 30°, 45°, 60°, and 90° (and their multiples).

<div align="center">Apr 19-9:13 PM</div>

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Consider an equilateral triangle with a side length of 2.
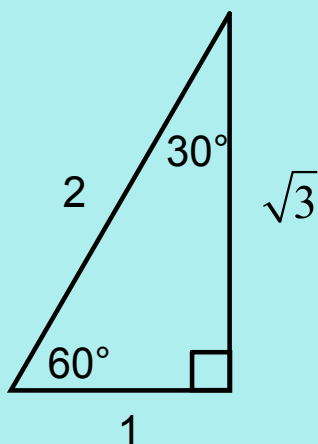


<div align="center">May 1-7:53 PM</div>

Now consider only ½ of the triangle.

$$a^2 + b^2 = c^2$$
$$1^2 + b^2 = 2^2$$
$$1 + b^2 = 4$$
$$b^2 = 3$$
$$b = \sqrt{3}$$

May 1-7:53 PM

Summary:

1. The 30°-60°-90° Triangle

$$\sin 30° = \frac{1}{2} \qquad \sin 60° = \frac{\sqrt{3}}{2}$$
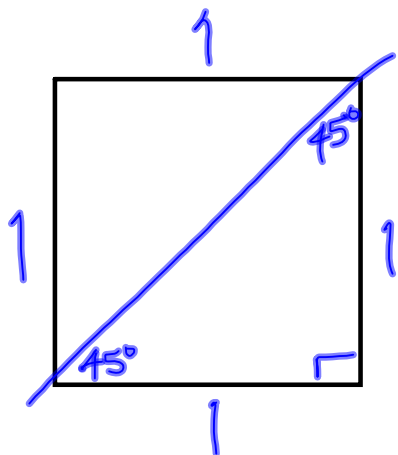
$$\cos 30° = \frac{\sqrt{3}}{2} \qquad \cos 60° = \frac{1}{2}$$

$$\tan 30° = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \qquad \tan 60° = \sqrt{3}$$

$$= \frac{\sqrt{3}}{3}$$

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Consider a square of side length 1.

1

45°

1                    1

45°

1

Now consider only ½ of the triangle.

?                    45°

45°                    1

1

isoceles

$$1^2 + 1^2 = hyp^2$$

$$1 + 1 = hyp^2$$

$$2 = hyp^2$$

$$\sqrt{2} = hyp$$

Summary:

2. The 45°-45°-90° Triangle

$$\sin 45° = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \qquad \cos 45° = \frac{1}{\sqrt{2}}$$

$$= \frac{\sqrt{2}}{2} \qquad\qquad\qquad = \frac{\sqrt{2}}{2}$$

$$\tan 45° = 1$$

(Triangle diagram: legs labeled $1$ and $1$, hypotenuse $\sqrt{2}$, angles 45°, 45°, and right angle.)

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What about right angles?

(Coordinate axes labeled $(0,1)$, $(-1,0)$, $(1,0)$, $(0,-1)$.)

$$\sin 0° = \frac{0}{1} = 0$$
$$\cos 0° = \frac{1}{1} = 1$$
$$\tan 0° = \frac{0}{1} = 0$$

$$\sin 90° = \frac{1}{1} = 1$$
$$\cos 90° = \frac{0}{1} = 0$$
$$\tan 90° = \frac{1}{0} = \text{und.}$$

$$\sin 180° = \frac{0}{1} = 0$$
$$\cos 180° = \frac{-1}{1} = -1$$
$$\tan 180° = \frac{0}{-1} = 0$$

$$\sin 270° = \frac{-1}{1} = -1$$
$$\cos 270° = \frac{0}{1} = 0$$
$$\tan 270° = \frac{-1}{0} = \text{und}$$

$$\sin 360° = 0$$
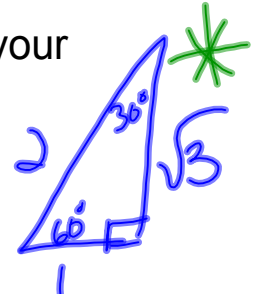$$\cos 360° = 1$$
$$\tan 360° = 0$$

May 3-9:34 AM

**4**

Ex.1 Evaluate using exact values and express your answer in reduced form.

(a) $\sin 45°$

$\sin 45° = \frac{o}{h}$

$= \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

(b) $\cos^2 60°$

$(\cos 60°)^2 = \left(\frac{a}{h}\right)^2$

$= \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

(c) $\cos 45° + \sin 30°$

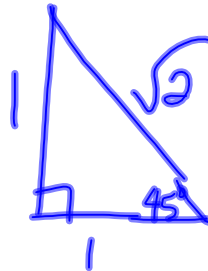$\frac{1}{\sqrt{2}} + \frac{1}{2}$

$= \frac{\sqrt{2}}{2} + \frac{1}{2} = \frac{\sqrt{2}+1}{2}$

(d) $2\tan 60° - 3\tan 30°$

$2\left(\frac{\sqrt{3}}{1}\right) - 3\left(\frac{1}{\sqrt{3}}\right)$

$= 2\sqrt{3} - \frac{3}{\sqrt{3}}$
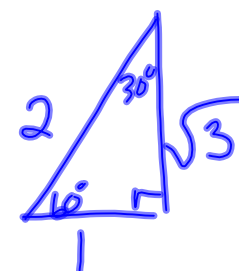
$= 2\sqrt{3} - \sqrt{3}$

$= \sqrt{3}$

Apr 28-11:19 PM

Ex.2 Determine the angle, $0° \le \theta \le 360°$

(a) $\sin\theta = \frac{1}{2} = \frac{opp}{hyp}$  $(30,60,90)$

$\frac{S|A}{T|C}$   $\theta = 30°, 150°$

$(180°-30°)$

(b) $\tan\theta = 1$   $(45,45,90)$

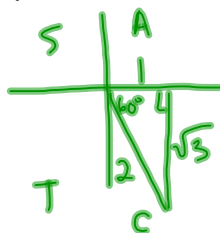$\frac{S|A}{T|C}$ $\theta = 45°, 225°$
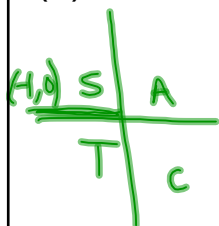
$(180°+45°)$

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Ex.3 Evaluate

(a) $\sin 315°$



$$\sin 315° = \frac{-1}{\sqrt{2}}$$

(b) $\tan 300°$



$$\tan 300° = -\sqrt{3}$$

(c) $\cos 180°$



$$\cos 180° = -1$$

(d) $\tan 270°$



$$\tan 270° = \frac{-1}{0} = und$$

May 2-9:50 PM

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Assigned Work:

Complete Handout Questions,
p.348 # 3

Apr 21-12:17 AM