Recall:

In the x-y plane, trig ratios are expressed in terms of x, y, and r.

$\sin\theta = \dfrac{y}{r}$ $\quad\cos\theta = \dfrac{x}{r}$ $\quad\tan\theta = \dfrac{y}{x}$ $\quad$ where: $r^2 = x^2 + y^2$

An identity is an equation which is always true for all values of the variable.

Quotient Identity: $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

Pythagorean Identity: $\sin^2\theta + \cos^2\theta = 1$

Apr 19-9:13 PM

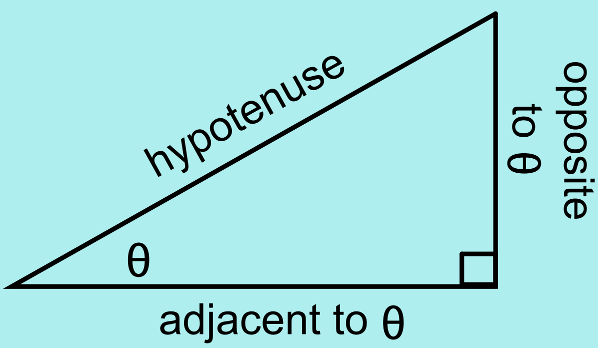
Recall:

For any angle of interest (θ), there are three (3) primary trigonometric ratios.

sine of $\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$

cosine of $\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$

tangent of $\theta = \dfrac{\text{opposite}}{\text{adjacent}}$



S o h C a h T o a

Apr 25-9:54 PM

Trigonometric Identities (continued)

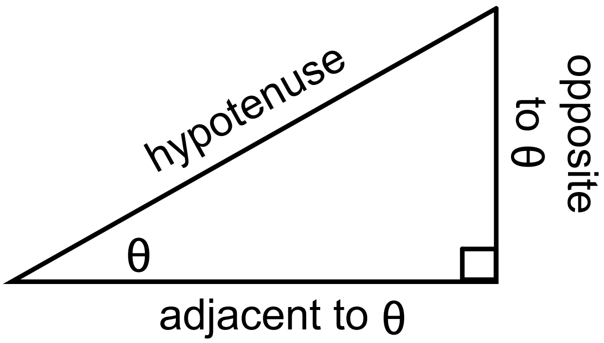
May 8/2012

The reciprocal identities are defined as follows:

$$\text{cosecant of } \theta = \frac{\text{hypotenuse}}{\text{opposite}}$$

$$\text{secant of } \theta = \frac{\text{hypotenuse}}{\text{adjacent}}$$

$$\text{cotangent of } \theta = \frac{\text{adjacent}}{\text{opposite}}$$



They are the reciprocals of the fundamental ratios:

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Apr 19-9:13 PM

Using the reciprocal identities, consider dividing $\sin^2\theta + \cos^2\theta = 1$ by $\cos^2\theta$.

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$\tan^2\theta = \sec^2\theta - 1$$

$$1 = \sec^2\theta - \tan^2\theta$$

May 1-7:53 PM

Using the reciprocal identities, consider dividing $\sin^2\theta + \cos^2\theta = 1$ by $\sin^2\theta$.

$$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$$1 + \cot^2\theta = \csc^2\theta$$

$$\cot^2\theta = \csc^2\theta - 1$$

$$1 = \csc^2\theta - \cot^2\theta$$

May 1-7:53 PM

Summary:

Quotient Identity:

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

Pythagorean Identity:

$$\sin^2\theta + \cos^2\theta = 1$$

Reciprocal Identities:

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Other Useful Identities:

$$\tan^2\theta + 1 = \sec^2\theta \qquad \cot^2\theta + 1 = \csc^2\theta$$

May 1-7:56 PM

Ex.1 Prove $\csc\theta = \dfrac{\cot\theta}{\cos\theta}$ (#1 from WS 3.3)

$$RS = \frac{\cot\theta}{\cos\theta}$$

$$= \frac{\frac{1}{\tan\theta}}{\frac{\cos\theta}{1}}$$

$$= \frac{1}{\tan\theta} \cdot \frac{1}{\cos\theta}$$

$$= \frac{1}{\frac{\sin\theta}{\cos\theta}} \cdot \frac{1}{\cos\theta}$$

$$= \frac{\cos\theta}{\sin\theta} \cdot \frac{1}{\cos\theta}$$

$$= \frac{1}{\sin\theta}$$

$$= \csc\theta$$

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \times \frac{d}{c}$$

Apr 28-11:19 PM

Ex.2 Prove $\csc^4 x - \cot^4 x = \csc^2 x + \cot^2 x$
(#5 from WS 3.3)

$$LS = \underbrace{(\csc^2 x - \cot^2 x)}_{=1?}\underbrace{(\csc^2 x + \cot^2 x)}_{\text{final answer}}$$

YES!

$$LS = \csc^2 x + \cot^2 x$$

$$\cot^2\theta + 1 = \csc^2\theta$$

Apr 28-11:19 PM

Assigned Work:

WS 3.3 # ~~2, 3, 4, 6, 9, 11, 12, 14, 15~~

ALL!

tricky: 7, 10, 11, 13, 16

16, 7, 9, 15, 11

Apr 21-12:17 AM

7. $(1-\tan\theta)^2 = \sec^2\theta - 2\tan\theta$

$$RS = \underbrace{\sec^2\theta}_{\downarrow} - 2\tan\theta$$
$$= \tan^2\theta + 1 - 2\tan\theta$$
$$= \tan^2\theta - 2\tan\theta + 1$$
$$x^2 - 2x + 1$$
$$= (x-1)(x-1)$$
$$= (\tan\theta - 1)(\tan\theta - 1)$$
$$= (\tan\theta - 1)^2$$

May 9-1:56 PM

9. $\frac{\cot w}{\cos w} + \frac{\sec w}{\cot w} = (\sec^2 w)(\csc w)$

$$LS = \frac{\frac{\cos w}{\sin w}}{\frac{\cos w}{1}} + \frac{\frac{1}{\cos w}}{\frac{\cos w}{\sin w}}$$

$$= \frac{\cos w}{\sin w} \cdot \frac{1}{\cos w} + \frac{1}{\cos w} \cdot \frac{\sin w}{\cos w}$$

$$= \frac{\cancel{\cos w}^{1}}{\sin w \cancel{\cos w}_{1}} + \frac{\sin w}{\cos^2 w}$$

$$= \frac{\cos^2 w + \sin^2 w}{\sin w \cos^2 w}$$

$$= \frac{1}{\sin w \cos^2 w}$$

$$= \sec^2 w \csc w$$

May 9-1:59 PM

11. $\sec y - \tan y \sin y = \cos y$

$$LS = \frac{1}{\cos y} - \frac{\sin y}{\cos y} \cdot \frac{\sin y}{1}$$

$$= \frac{1}{\cos y} - \frac{\sin^2 y}{\cos y}$$

$$= \frac{1 - \sin^2 y}{\cos y}$$

$$= \frac{\cos^2 y}{\cos y}$$

$$= \cos y$$

May 9-2:07 PM

16. $\frac{1-\tan^2 w}{1+\tan^2 w} = 2\cos^2 w - 1$

$LS = \dfrac{1 - \frac{\sin^2 w}{\cos^2 w}}{\sec^2 w}$

$= \dfrac{\frac{\cos^2 w - \sin^2 w}{\cos^2 w}}{\frac{1}{\cos^2 w}}$

$= \frac{\cos^2 w - \sin^2 w}{\cos^2 w} \cdot \frac{\cos^2 w}{1}$

$= \cos^2 w - \sin^2 w$

$= \cos^2 w - (1 - \cos^2 w)$

$= \cos^2 w - 1 + \cos^2 w$

$= 2\cos^2 w - 1$

May 9-2:08 PM