Trig Identities worksheet 3.3 name:

Prove each identity:

1. $\csc\theta = \dfrac{\cot\theta}{\cos\theta}$

$RS = \dfrac{\frac{\cos\theta}{\sin\theta}}{\cos\theta}$

$= \dfrac{\cos\theta}{\sin\theta} \times \dfrac{1}{\cos\theta}$

$= \dfrac{1}{\sin\theta}$

$= \csc\theta$

2. $\dfrac{1}{\sec^2 x} + \dfrac{1}{\csc^2 x} = 1$

$LS = \cos^2\theta + \sin^2\theta$

$= 1.$

3. $\csc^2 y \tan^2 y - 1 = \tan^2 y$

$LS = \dfrac{1}{\sin^2 y} \times \dfrac{\sin^2 y}{\cos^2 y} - 1$

$= \dfrac{1}{\cos^2 y} - 1$

$= \sec^2 y - 1$

$= \tan^2 y$

4. $\dfrac{\sec\theta}{\cos\theta} - \dfrac{\tan\theta}{\cot\theta} = 1$

$LS = \dfrac{\frac{1}{\cos\theta}}{\cos\theta} - \dfrac{\frac{\sin\theta}{\cos\theta}}{\frac{\cos\theta}{\sin\theta}}$

$= \dfrac{1}{\cos^2\theta} - \dfrac{\sin^2\theta}{\cos^2\theta}$

$= \dfrac{1-\sin^2\theta}{\cos^2\theta}$

$= \dfrac{\cos^2\theta}{\cos^2\theta}$

$\rightarrow LS = 1.$

5. $\csc^4 x - \cot^4 x = \csc^2 x + \cot^2 x$

$LS = (\csc^2 x - \cot^2 x)(\csc^2 x + \cot^2 x)$

$= \left(\dfrac{1}{\sin^2 x} - \dfrac{\cos^2 x}{\sin^2 x}\right)(\csc^2 x + \cot^2 x)$

$= \left(\dfrac{1-\cos^2 x}{\sin^2 x}\right)(\csc^2 x + \cot^2 x)$

$= \left(\dfrac{\sin^2 x}{\sin^2 x}\right)(\csc^2 x + \cot^2 x)$

$= \csc^2 x + \cot^2 x$

6. $\sec^4 y - \tan^4 y = \tan^2 y + \sec^2 y$

$LS = (\sec^2 y - \tan^2 y)(\sec^2 y + \tan^2 y)$

$= \left(\dfrac{1}{\cos^2 y} - \dfrac{\sin^2 y}{\cos^2 y}\right)(\tan^2 y + \sec^2 y)$

$= \left(\dfrac{1-\sin^2 y}{\cos^2 y}\right)(\tan^2 y + \sec^2 y)$

$= \left(\dfrac{\cos^2 y}{\cos^2 y}\right)(\tan^2 y + \sec^2 y)$

$= \tan^2 y + \sec^2 y$

7. $(1-\tan\theta)^2 = \sec^2\theta - 2\tan\theta$

$RS = \sec^2\theta - 2\tan\theta$

$= \tan^2\theta + 1 - 2\tan\theta$

$= \tan^2\theta - 2\tan\theta + 1$

$a^2 - 2a + 1$

$= (\tan\theta - 1)^2$

$= (1-\tan\theta)^2$

8. $(1-\sin^2 x)(1+\tan^2 x) = 1$

$LS = (1-\sin^2 x)(\sec^2 x)$

$= (1-\sin^2 x)\left(\dfrac{1}{\cos^2 x}\right)$

$= \dfrac{1}{\cos^2 x} - \dfrac{\sin^2 x}{\cos^2 x}$

$= \dfrac{\cos^2 x}{\cos^2 x}$

$= 1.$

9. $\dfrac{\cot w}{\cos w}+\dfrac{\sec w}{\cot w}=\sec^2 w\csc w$

$LS=\dfrac{\cos w}{\sin w}\cdot\dfrac{1}{\cos w}+\dfrac{1}{\cos w}\cdot\dfrac{1}{\frac{\cos w}{\sin w}}$

$=\dfrac{1}{\sin w}+\dfrac{\sin w}{\cos^2 w}$

$=\dfrac{\cos^2 w+\sin^2 w}{\sin w\cos^2 w}$

$=\dfrac{1}{\sin w\cos^2 w}$

$=\csc w\sec^2 w$

$=\sec^2 w\csc w$

10. $2\sin^2 y-1=1-2\cos^2 y$

$LS=2(1-\cos^2 y)-1$

$=2-2\cos^2 y-1$

$=1-2\cos^2 y$

11. $\sec y-\tan y\sin y=\cos y$

$LS=\dfrac{1}{\cos y}-\dfrac{\sin y}{\cos y}\cdot\sin y$

$=\dfrac{1-\sin^2 y}{\cos y}$

$=\dfrac{\cos^2 y}{\cos y}$

$=\cos y$

12. $\left(1-\cos^2\theta\right)\left(\cot^2\theta+1\right)=1$

$LS=(1-\cos^2\theta)\left(\dfrac{\cos^2\theta}{\sin^2\theta}+1\right)$

$=(1-\cos^2\theta)\left(\dfrac{\cos^2\theta+\sin^2\theta}{\sin^2\theta}\right)$

$=\sin^2\theta\left(\dfrac{1}{\sin^2\theta}\right)$

$=1$

13. $\dfrac{1-\sin^2\theta}{1+\tan^2\theta}=\cos^4\theta$

$LS=\dfrac{\cos^2\theta}{1+\frac{\sin^2\theta}{\cos^2\theta}}$

$=\dfrac{\cos^2\theta}{\frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta}}$

$=\dfrac{\cos^4\theta}{\cos^2\theta+\sin^2\theta}$

$=\cos^4\theta$

14. $\dfrac{\sin t}{\csc t}+\dfrac{\cos t}{\sec t}=1$

$LS=\dfrac{\sin t}{\frac{1}{\sin t}}+\dfrac{\cos t}{\frac{1}{\cos t}}$

$=\sin^2 t+\cos^2 t$

$=1$

15. $\dfrac{\sin y+\tan y}{1+\sec y}=\sin y$

$LS=\dfrac{\sin y+\frac{\sin y}{\cos y}}{1+\frac{1}{\cos y}}$

$=\dfrac{\frac{\sin y\cos y+\sin y}{\cos y}}{\frac{\cos y+1}{\cos y}}$

$=\dfrac{\sin y\cos y+\sin y}{\cos y}\times\dfrac{\cos y}{\cos y+1}$

$=\dfrac{\sin y(\cos y+1)}{\cos y+1}$

$=\sin y$

16. $\dfrac{1-\tan^2 w}{1+\tan^2 w}=2\cos^2 w-1$

$LS=\dfrac{1-\frac{\sin^2 w}{\cos^2 w}}{1+\frac{\sin^2 w}{\cos^2 w}}$

$=\dfrac{\frac{\cos^2 w-\sin^2 w}{\cos^2 w}}{\frac{\cos^2 w+\sin^2 w}{\cos^2 w}}$

$=\dfrac{\cos^2 w-\sin^2 w}{\cos^2 w}\cdot\dfrac{\cos^2 w}{1}$

$=\cos^2 w-\sin^2 w$

$=\cos^2 w-(1-\cos^2 w)$

$=2\cos^2 w-1$