

Prove each identity:

$$\begin{aligned}
 1. \quad \csc \theta &= \frac{\cot \theta}{\cos \theta} & RS &= \frac{\frac{\cos \theta}{\sin \theta}}{\cos \theta} \\
 & & &= \frac{\cos \theta}{\sin \theta} \times \frac{1}{\cos \theta} \\
 & & &= \frac{1}{\sin \theta} \\
 & & &= \csc \theta
 \end{aligned}$$

$$\begin{aligned}
 2. \quad \frac{1}{\sec^2 x} + \frac{1}{\csc^2 x} &= 1 \\
 LS &= \cos^2 \theta + \sin^2 \theta \\
 &= 1.
 \end{aligned}$$

$$\begin{aligned}
 3. \quad \csc^2 y \tan^2 y - 1 &= \tan^2 y \\
 LS &= \frac{1}{\sin^2 y} \times \frac{\sin^2 y}{\cos^2 y} - 1 \\
 &= \frac{1}{\cos^2 y} - 1 \\
 &= \sec^2 y - 1 \\
 &= \tan^2 y
 \end{aligned}$$

$$\begin{aligned}
 4. \quad \frac{\sec \theta}{\cos \theta} - \frac{\tan \theta}{\cot \theta} &= 1 \\
 LS &= \frac{1}{\cos \theta} - \frac{\frac{\sin \theta}{\cos \theta}}{\frac{\cos \theta}{\sin \theta}} \\
 &= \frac{1}{\cos^2 \theta} - \frac{\sin^2 \theta}{\cos^2 \theta} \\
 &= \frac{1 - \sin^2 \theta}{\cos^2 \theta} \\
 &= \frac{\cos^2 \theta}{\cos^2 \theta} \rightarrow LS = 1.
 \end{aligned}$$

$$\begin{aligned}
 5. \quad \csc^4 x - \cot^4 x &= \csc^2 x + \cot^2 x \\
 LS &= (\csc^2 x - \cot^2 x)(\csc^2 x + \cot^2 x) \\
 &= \left(\frac{1}{\sin^2 x} - \frac{\cos^2 x}{\sin^2 x} \right) (\csc^2 x + \cot^2 x) \\
 &= \left(\frac{1 - \cos^2 x}{\sin^2 x} \right) (\csc^2 x + \cot^2 x) \\
 &= \left(\frac{\sin^2 x}{\sin^2 x} \right) (\csc^2 x + \cot^2 x) \\
 &= \csc^2 x + \cot^2 x
 \end{aligned}$$

$$\begin{aligned}
 6. \quad \sec^4 y - \tan^4 y &= \tan^2 y + \sec^2 y \\
 LS &= (\sec^2 y - \tan^2 y)(\sec^2 y + \tan^2 y) \\
 &= \left(\frac{1}{\cos^2 y} - \frac{\sin^2 y}{\cos^2 y} \right) (\tan^2 y + \sec^2 y) \\
 &= \left(\frac{1 - \sin^2 y}{\cos^2 y} \right) (\tan^2 y + \sec^2 y) \\
 &= \left(\frac{\cos^2 y}{\cos^2 y} \right) (\tan^2 y + \sec^2 y) \\
 &= \tan^2 y + \sec^2 y
 \end{aligned}$$

$$\begin{aligned}
 7. \quad (1 - \tan \theta)^2 &= \sec^2 \theta - 2 \tan \theta \\
 RS &= \sec^2 \theta - 2 \tan \theta \\
 &= \tan^2 \theta + 1 - 2 \tan \theta \\
 &= \tan^2 \theta - 2 \tan \theta + 1 \\
 &\quad a^2 - 2a + 1 \\
 &= (\tan \theta - 1)^2 \\
 &= (1 - \tan \theta)^2
 \end{aligned}$$

$$\begin{aligned}
 8. \quad (1 - \sin^2 x)(1 + \tan^2 x) &= 1 \\
 LS &= (1 - \sin^2 x)(\sec^2 x) \\
 &= (1 - \sin^2 x) \left(\frac{1}{\cos^2 x} \right) \\
 &= \frac{1}{\cos^2 x} - \frac{\sin^2 x}{\cos^2 x} \\
 &= \frac{\cos^2 x}{\cos^2 x} \\
 &= 1.
 \end{aligned}$$

9. $\frac{\cot w}{\cos w} + \frac{\sec w}{\cot w} = \sec^2 w \csc w$

$$\begin{aligned} \text{LS} &= \frac{\cos w}{\sin w} \cdot \frac{1}{\cos w} + \frac{1}{\cos w} \cdot \frac{1}{\frac{\cos w}{\sin w}} \\ &= \frac{1}{\sin w} + \frac{\sin w}{\cos^2 w} \\ &= \frac{\cos^2 w + \sin^2 w}{\sin w \cos^2 w} \rightarrow \begin{aligned} &= \csc w \sec^2 w \\ &= \sec^2 w \csc w \end{aligned} \\ &= \frac{1}{\sin w \cos^2 w} \end{aligned}$$

10. $2\sin^2 y - 1 = 1 - 2\cos^2 y$

$$\begin{aligned} \text{LS} &= 2(1 - \cos^2 y) - 1 \\ &= 2 - 2\cos^2 y - 1 \\ &= 1 - 2\cos^2 y \end{aligned}$$

11. $\sec y - \tan y \sin y = \cos y$

$$\begin{aligned} \text{LS} &= \frac{1}{\cos y} - \frac{\sin y}{\cos y} \cdot \sin y \\ &= \frac{1 - \sin^2 y}{\cos y} \\ &= \frac{\cos^2 y}{\cos y} \\ &= \cos y \end{aligned}$$

12. $(1 - \cos^2 \theta)(\cot^2 \theta + 1) = 1$

$$\begin{aligned} \text{LS} &= (1 - \cos^2 \theta) \left(\frac{\cot^2 \theta}{\sin^2 \theta} + 1 \right) \\ &= (1 - \cos^2 \theta) \left(\frac{\cos^2 \theta + \sin^2 \theta}{\sin^2 \theta} \right) \\ &= \sin^2 \theta \left(\frac{1}{\sin^2 \theta} \right) \\ &= 1 \end{aligned}$$

13. $\frac{1 - \sin^2 \theta}{1 + \tan^2 \theta} = \cos^4 \theta$

$$\begin{aligned} \text{LS} &= \frac{\cos^2 \theta}{1 + \frac{\sin^2 \theta}{\cos^2 \theta}} \rightarrow \begin{aligned} &= \frac{\cos^4 \theta}{\cos^2 \theta + \sin^2 \theta} \\ &= \cos^4 \theta \end{aligned} \\ &= \frac{\cos^2 \theta}{\frac{\cos^2 \theta + \sin^2 \theta}{\cos^2 \theta}} \end{aligned}$$

14. $\frac{\sin t}{\csc t} + \frac{\cos t}{\sec t} = 1$

$$\begin{aligned} \text{LS} &= \frac{\sin t}{\frac{1}{\sin t}} + \frac{\cos t}{\frac{1}{\cos t}} \\ &= \sin^2 t + \cos^2 t \\ &= 1 \end{aligned}$$

15. $\frac{\sin y + \tan y}{1 + \sec y} = \sin y$

$$\begin{aligned} \text{LS} &= \frac{\sin y + \frac{\sin y}{\cos y}}{1 + \frac{1}{\cos y}} \rightarrow \begin{aligned} &= \frac{\sin y (\cos y + 1)}{\cos y + 1} \\ &= \sin y \end{aligned} \\ &= \frac{\sin y \cos y + \sin y}{\frac{\cos y + 1}{\cos y}} \\ &= \frac{\sin y \cos y + \sin y}{\cos y} \cdot \frac{\cos y}{\cos y + 1} \end{aligned}$$

16. $\frac{1 - \tan^2 w}{1 + \tan^2 w} = 2\cos^2 w - 1$

$$\begin{aligned} \text{LS} &= \frac{1 - \frac{\sin^2 w}{\cos^2 w}}{1 + \frac{\sin^2 w}{\cos^2 w}} \rightarrow \begin{aligned} &= \frac{\cos^2 w - \sin^2 w}{\cos^2 w + \sin^2 w} \\ &= \frac{\cos^2 w - \sin^2 w}{1} \\ &= \cos^2 w - \sin^2 w \\ &= \cos^2 w - (1 - \cos^2 w) \\ &= 2\cos^2 w - 1 \end{aligned} \\ &= \frac{\cos^2 w - \sin^2 w}{\cos^2 w} \cdot \frac{\cos^2 w}{1} \end{aligned}$$