# Solving Linear Trigonometric equations

To solve a trigonometric equation means to find the angle value(s) that satisfy the given equation.

recall: solving linear equations

Solve for x:

$2x + 3 = 4x - 5$

# Solving Linear Trigonometric equations

May 9/2012

To solve a trigonometric equation means to find the angle value(s) that satisfy the given equation.

Steps:

- the equation should involve only one trigonometric ratio

- isolate the trigonometric ratio

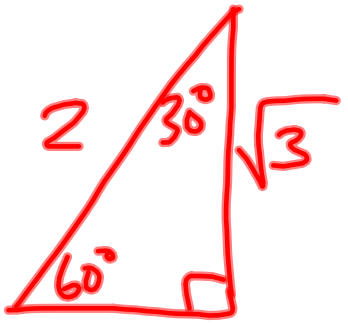
- solve for the related acute angle (using positive ratio)

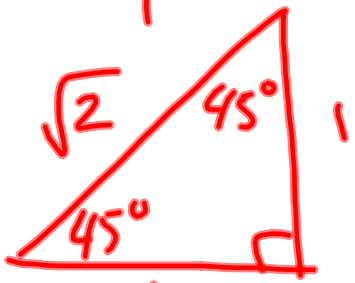
- use the actual sign of the ratio to determine the quadrant(s) for your answer [CAST]

- determine the angles, within your chosen quadrants, using the related acute angle

Ex: 1) Solve each equation for $0° < \theta \leq 360°$.

a) $\sin\theta = \frac{\sqrt{2}}{2}$




③ RAA $= \sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$

$= 45°$

④ S | A
T | C  Q1 or Q2

⑤ Q1 : $\theta = 45°$

Q2 : $\theta = 135°$ $(180° - 45°)$

$\sin 45° = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{\sqrt{2}}{2}$

b) $\cos\theta = \frac{-1}{2}$

using positive ratio

③ $RAA = \cos^{-1}\left(\frac{1}{2}\right)$

$= 60°$

④ S | A / T | C  Q2 & Q3

⑤ Q2 : $\theta = 120°$  Q3 : $\theta = 240°$

c) $\sin\theta + \sqrt{3} = -\sin\theta$

② $+\sin\theta - \sqrt{3}$ $+\sin\theta - \sqrt{3}$

$$\frac{2\sin\theta}{2} = -\frac{\sqrt{3}}{2}$$

$$\sin\theta = -\frac{\sqrt{3}}{2}$$

③ $RAA = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$= 60°$

④ S | A
T | C  Q3 & Q4

⑤ Q3: $\theta = 240°$

Q4: $\theta = 300°$

d) $5\tan\theta = 2\tan\theta + 7$

② $3\tan\theta = 7$

$\tan\theta = \frac{7}{3}$

③ $RAA = \tan^{-1}\left(\frac{7}{3}\right)$

$\doteq 66.8°$

④


Q1 & Q3

⑤ Q1: $\theta \doteq 66.8°$

Q3: $\theta = 180° + 66.8°$

$\doteq 246.8°$

e) $\cos(2\theta) = \frac{\sqrt{3}}{2}$

Let $x = 2\theta$

$\cos x = \frac{\sqrt{3}}{2}$

③ $RAA = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$= 30°$

④ S A T C Q1 & Q4

⑤

$x = 30°$ | $x = 330°$

$2\theta = 30°$ | $2\theta = 330°$

$\theta = 15°$ | $\theta = 165°$

$x = 30° + 360°$ | $x = 690°$

$x = 390°$ | $2\theta = 690°$

$2\theta = 390°$ | $\theta = 345°$

$\theta = 195°$

f) ~~$3\sin 2\theta + 3 = 5$~~

$11\sin\left(\frac{1}{2}\theta\right) + 3 = -5$

let $x = \frac{1}{2}\theta$

$11\sin x + 3 = -5$

$11\sin x = -8$

$\sin x = -\frac{8}{11}$

③ $RAA = \sin^{-1}\left(\frac{8}{11}\right)$

$\doteq 46.7^\circ$

④ S | A

T | C  Q3 & Q4

⑤ $x \doteq 180^\circ + 46.7^\circ$

$\doteq 226.7^\circ$

$x \doteq 360^\circ - 46.7^\circ$

$\doteq 313.3^\circ$

$\frac{1}{2}\theta \doteq 226.7^\circ$

$\theta \doteq 453.4^\circ$

$\frac{1}{2}\theta \doteq 313.3^\circ$

$\theta \doteq 626.6^\circ$

but $0^\circ < \theta \leq 360^\circ$

$\therefore$ no solution

Assigned Work:

p.408 # 1ace, 2adf, 5ab, 14agi

$0 < \theta \leq 360°$

$2\pi \text{ rad} = 360°$

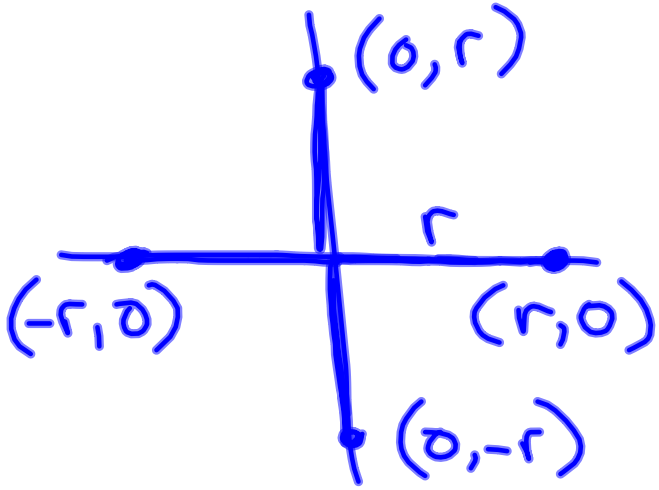
14(a) $\sin 2x = 1$

Let $2x = \theta$

$\sin\theta = 1$

$RAA = \sin^{-1}(1)$

$= 90°$




$\sin\theta = \frac{y}{r}$

$\theta = 90° \rightarrow$ $\theta = 90° + 360°$

$2x = 90°$ $= 450°$

$x = 45°$ $2x = 450°$

$x = 225°$

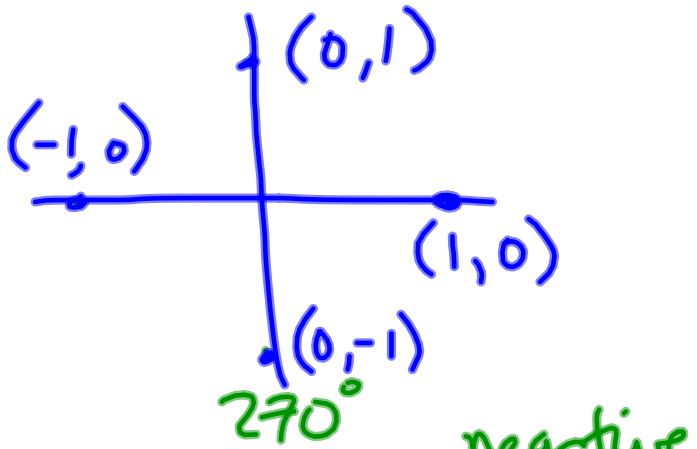
2(a) $\sin x + 1 = 0$

$\sin x = -1$

RAA $= \sin^{-1}(1)$

$= 90°$

$x = 270°$




$\sin\theta = \frac{y}{r}$ ← $y$: negative; ← $r$: always positive