Periodic Functions

A periodic function is a function that repeats its behaviour identically at regular intervals. The pattern that repeats is called a **cycle**.

The length of one cycle (measured along the horizontal axis) is called the **period** of the function.

The horizontal line halfway between the maximum and minimum values is called the **axis of the curve** (a line dividing the function in two halves). This line has an equation of $y = \frac{Maximum + minimum}{2}$. Recall: the maximum value is the largest y value of the function and the minimum value is the lowest y value of the function.

The vertical distance from the axis of the curve to the maximum (or the vertical distance from the axis of the curve to the minimum) is called the **amplitude**. The amplitude, a, can be determined with the equation $a = \frac{Maximum - minimum}{2}$.

Ex: 1.
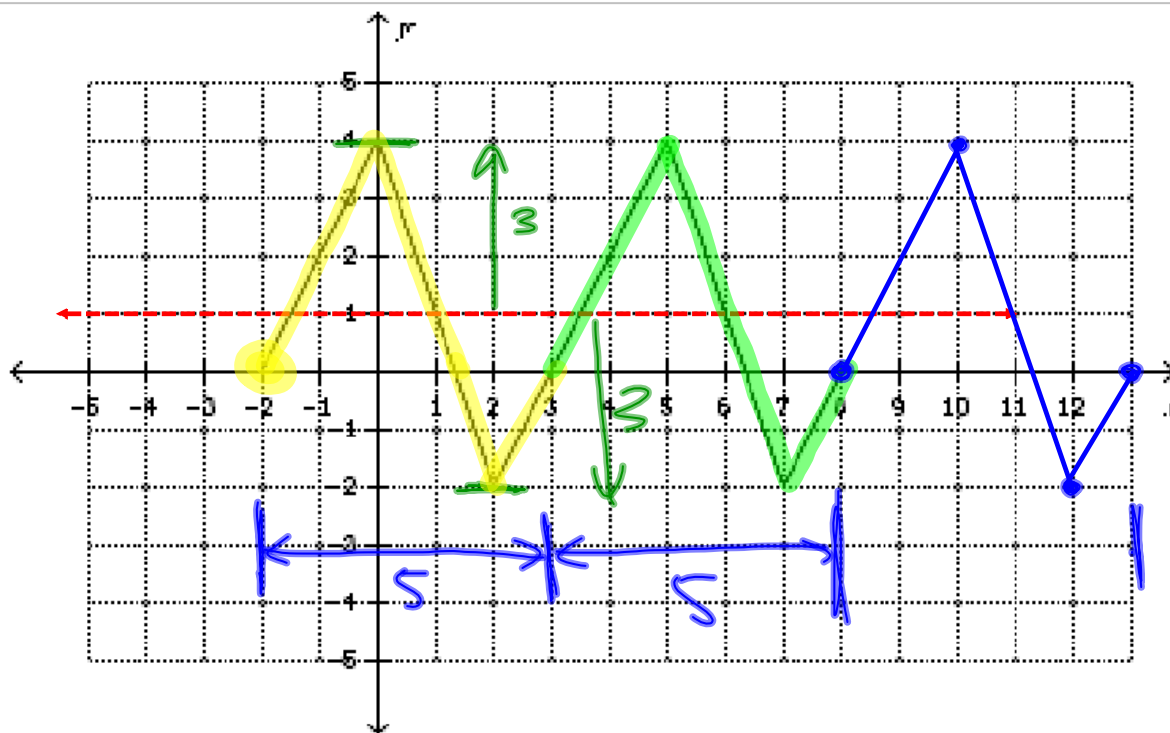
a) How many cycles are shown in the graph?

b) What is the *period*?

c) Draw an additional cycle on the graph.

c) For the given periodic function f, state the:

i) maximum value

4

ii) minimum value

-2

iii) amplitude

$$\frac{4-(-2)}{2}$$

$$= \frac{6}{2}$$

$$= 3$$

d) Suppose the periodic behaviour continues. Then, what is the value of:

i) $f(12)$ ii) $f(-4)$ iii) $f(35)$ iv) $f(71)$

period 5

$-4+5=1$

$\therefore f(-4) = f(1)$

$f(1) = 1.$

find x-value that matches 35

① subtract 5 multiple times until reaching an x-value on graph

② divide by period (5) and look at remainder

$\frac{35}{5} = 7$ R.O
remainder

$\therefore f(35) = f(0)$
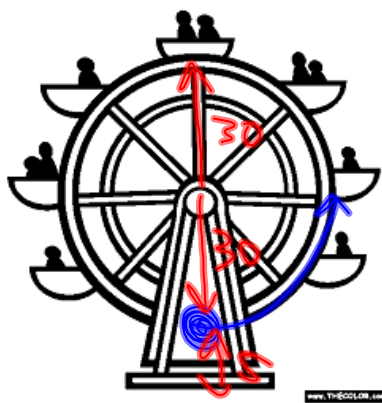
(d) $f(71)$

$= f(1)$

$\frac{71}{5} = 14.2$

$R = 0.2 \times 5$

$= 1$

period $= 11$ $f(614)$

$$\frac{614}{11} \doteq 55.818\ldots$$

$$\begin{array}{r} -55 \ (\text{on calc}) \\ \hline 0.818\ldots\ldots \end{array}$$

$$\begin{array}{r} \times 11 \\ \hline 9 \end{array}$$
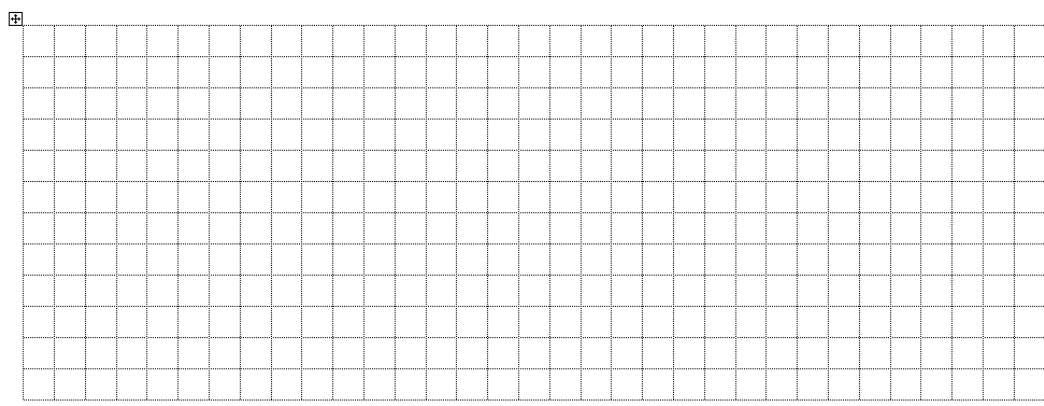
$$f(614) = f(9)$$

A ferris wheel has a diameter of 60 m, and stands 5 m above the ground. As it rotates, the height of a particular seat above the ground is changing. It takes 1 minute for the ferris wheel to make 1 complete revolution. What is the function of its height, relative to the time?

Time (min)	0	0.25	0.5	0.75	1.0
Height (m)	5	35	65	35	5
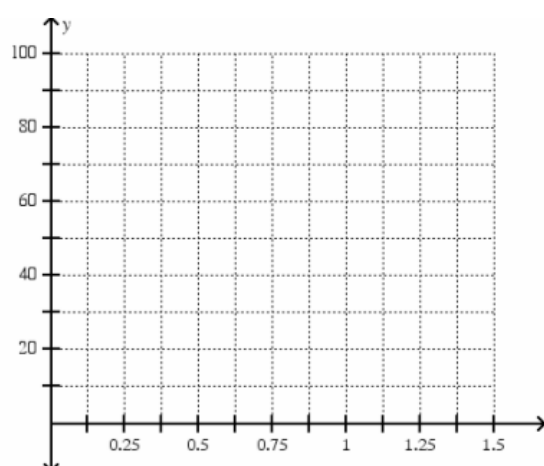
3. Sketch a graph of a periodic function with a period of 6 and amplitude of 4. Compare your graph with a classmate's. Are they the same? Different? Are you both correct?

HW: Pg.359 #1, 2, 4, 5, 7, 8, 10