MCR 3U **Course Review – June 2012**

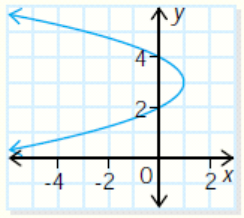
Functions and Transformations

1. For each relation,

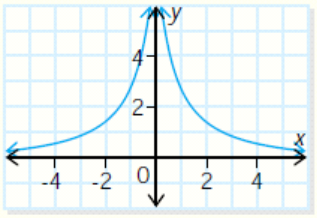
   i. identify the domain and range

   ii. tell whether it is a function or not. Justify your answer.

(a)



(b)



(c) $y = \sqrt{x - 5}$

(d) $y = 2(x - 3)^2 + 4$

2. For $f(x) = 2x^2 - 3x + 5$. Find

   a) $f(-2)$ b) $f\left(\frac{1}{3}\right)$ c) $f(0)$ d) $f(2x)$ e) a when $f(a) = 4$

3. Describe the transformations that must be made to the basic function to get the new function.

   a) $y = 3f(8 - 4x) - 5$ b) $y = -0.5(2)^{4x} + 1$ c) $y = \frac{1}{x-2} + 4$

   d) $y = -2\sqrt{5 - 10x} + 1$ e) $y = \frac{2}{3}\sin\left(\frac{x}{4} + 2\right) - 6$

4. For $g(x) = -3x^2 + 6x + 2$

   (a) graph g and g^{-1} on the same set of axes

   (b) determine the equation for g^{-1}

   (c) state restrictions on the domain or range of g so that its inverse is a function

   (d) assume the domain of g is $\{x \mid 0 \le x \le 5, x \in R\}$. Would the inverse be a function? Justify your answer.

Quadratics

5. Determine the standard form of the equation of the parabola with the following properties:

   a) a vertex at (3, –5) and passing through the point (–2, 10).

   b) zeros at $\frac{4 \pm \sqrt{8}}{2}$, reflected on the x –axis and vertically stretched by a factor of 2.

   c) in the same family as $y = 3x^2 + 6x + 3$

6. a) Determine the point(s) of intersection between $f(x) = 5x - 4$ and $g(x) = 3x^2 + 4x - 8$.
   b) Determine the equation(s) of the line(s) with a slope of -4 and that intersect $g(x) = 2x^2 - 8x + 1$ once, twice or never.

Radicals and Rational Functions

7. Simplify the following:

   a) $\sqrt{243}$ b) $2\sqrt{24}$ c) $6\sqrt{80} - 3\sqrt{125}$ d) $\dfrac{6}{\sqrt{3}}$ e) $\dfrac{1-\sqrt{2}}{3+\sqrt{2}}$

   f) $\left(2\sqrt{3} - \sqrt{5}\right)^2$ g) $\left(2\sqrt{3} - \sqrt{5}\right)\left(2\sqrt{3} + \sqrt{5}\right)$

8. Simplify. State any restrictions.

   (a) $\dfrac{5x^2 - 5}{x^2 - 4x - 5}$

   (b) $\dfrac{4x^4y}{3x^2y^4} \times \dfrac{-6x^3y^2}{10x^4}$

   (c) $\dfrac{2m^2 - m - 15}{m + 2} \times \dfrac{m^2 - m - 6}{m^2 - m + 9}$

   (d) $\dfrac{x^2 - y^2}{2x^2 - 8x} \times \dfrac{(x - y)^2}{2xy}$

   (e) $\dfrac{1}{x} - \dfrac{3}{y} + \dfrac{3x - y}{xy}$

   (f) $\dfrac{x + 2}{x^2 + 5x + 6} - \dfrac{x - 3}{x^2 - 3x - 10}$

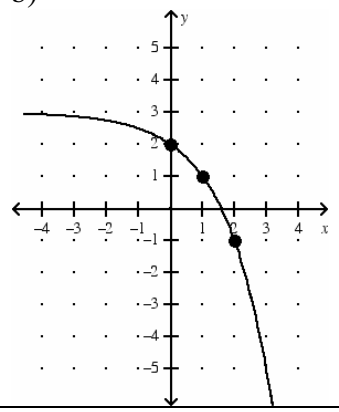
Exponential Functions

9. Determine the exponential function that relates to the following:

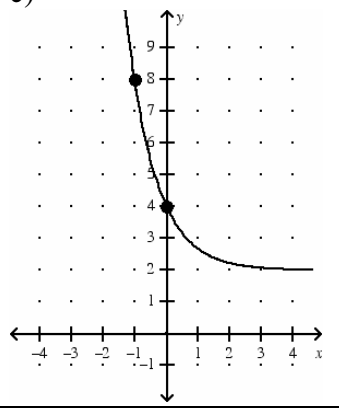
a)

Time, t, h	Number of Bacteria, N(t)
0	45 000 000
1	15 000 000
2	5 000 000
3	1 666 667
4	555 556
5	185 185
6	61 728

b)



c)



   d) The function $y = 4^x$ is reflected on the x-axis, vertically stretched by a factor of 3 and horizontally shifted to the right 5. The horizontal asymptote is $y = 3$ (on the left).

10. Land developers had to delay a housing development because the land had been previously used as a nuclear waste pit for Radium-D, an isotope. Health officials ordered them not to build until the isotope has decayed to $\dfrac{1}{\sqrt{2}}$ of its original mass. If the isotope has a half life of 22 years, will they be able to build in 10 years?

11. A certain bacterial strain divides every 0.5 h. If 500 bacteria are present in a culture, how many will there be after : a) 3 h? b) 6 h? c) n h?

12. Solve. $4^{x-2} = \dfrac{1}{64}$

13. Simplify using exponent laws:

a) -7^0 b) $\left(\frac{1}{4}\right)^{-2}$ c) $25^{m+1} \div 5^{m+1}$ d) $\left(\frac{4ac}{b^{-3}}\right)^{-2}$ e) $\frac{3^{-1}-4^{-1}}{12^{-1}}$ f) $16^{-\frac{3}{4}}$ g) $-25^{-\frac{3}{2}}$

h) $\frac{a^{\frac{1}{2}}a^{\frac{1}{3}}}{a^{\frac{1}{6}}}$ i) $\sqrt[5]{\frac{1}{32}}$ j) $\sqrt{\frac{(25x)^{\frac{1}{2}}\left(8x^2\right)^{\frac{1}{3}}}{40x^{\frac{1}{6}}}}$ k) $\left(\sqrt[5]{\frac{x^{\frac{1}{2}}\sqrt{x^3}}{\sqrt{x}}}\right)^2$

14. Simplify.

(a) $\frac{16^{\frac{3}{4}} - 27^{-\frac{2}{3}}}{4^{-\frac{1}{2}}}$ (b) $\left(\frac{27x^2y^{-5}}{64x^{-1}y^4}\right)^{\frac{2}{3}}$ (c) $\left(\frac{4}{x}\right)^{n-m}(2x^2)^m$

Periodic Functions and Trigonometry

15. Determine the value of each of the following to 4 decimal places:

a) sec 34° b) csc 325° c) cot 130°

16. Determine the value(s) of each angle correct to the nearest degree, $0^\circ \le \theta \le 360^\circ$:

a) sec θ = 2.5741 b) csc θ = –5.8452 c) cot θ = –0.4568

17. Determine the exact value(s) of the following:

a) sec 30° b) csc 300° c) cot 135°

18. What point on the unit circle corresponds to a rotation indicated below?

a) 30° b) 120° c) 225° d) 330° e) 270°

19. Solve. Round angles to the nearest degree.

a) $5\cos\theta - 4 = 0$ b) $3\sin^2\theta - 11\sin\theta - 4 = 0$

20. Prove that $(\tan x + \tan y)(1 - \cot x \cot y) + (\cot x + \cot y)(1 - \tan x \tan y) = 0$.

21. Prove each identity

(a) $\frac{2}{\sin^2\theta} = \frac{1}{1-\cos\theta} + \frac{1}{1+\cos\theta}$ (b) $\tan^2\theta + \cos^2\theta + \sin^2\theta = \frac{1}{\cos^2\theta}$

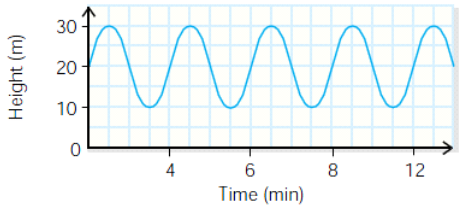
22. Determine the number of solutions each of the following triangles will have

a) $\angle A = 44.3°$, a= 11.5 m, and b = 7.7 m b) $\angle A = 29.3°$, b = 20.5 cm, and a = 12.8 cm

23. Solve triangle LMN, given $\angle L = 38°$, $l = 30\ cm$, and $n = 45\ cm$.

24. An airplane carrying smugglers leaves a private airfield at 06:00 and flies on a course of N40ºE at 200 km/h. The plane is detected by radar at the police airport, which is located 150 km Northwest of the private airfield. At 06:30 the police airplane leaves its airport with the intention of intercepting the smuggler at 08:30. At the time the smugglers are intercepted by the police airplane, how far away are the smugglers from the police airport? Round your answer to the nearest kilometer. Recall: *speed = dist ÷ time* and Northwest means N45ºW.

25. For the periodic function shown below identify the amplitude, the period, then determine a sine equation and a cosine equation for the function.



26. Sketch the graph of each function
    a) $f(x) = 3\sin x - 2$ b) $-2\cos(x - 30°)$

27. During high tide, at 12:06 a.m., the water in an inlet is 18 m deep. During low tide, which occurs 12 hours later, it is 11.5 m deep.
    a) Determine an expression for the water depth in the inlet, *t* hours after high tide.
    b) Sketch a graph of the function over a 24-h period.
    c) Determine the times on this day when the water is 15 m deep and 12 m deep.

Discrete

28. Determine the first six terms of the sequence in which $t_1 = 5$, $t_2 = 3$, and $t_n = 4t_{n-1} + 2t_{n-2}$.

29. For each sequence, find
    i. the general term
    ii. t_{25}
    iii. S_{20}

    (a) $-4, 5, 14, \ldots$
    (b) $3, 3.3, 3.63, 3.993, \ldots$

30. Determine the number of terms in each sequence.
    a) 3, 12, 48, …, 201 326 592 b) 18, 22, 26, …, 162

31. Determine
    a) The amount of $8500 will grow to if invested at 8%/a compounded quarterly for 10 years.
    b) The principal that must be invested now at 6%/a compounded annually to be worth $10 000 in 5 years.

32. A rare coin was bought for $1200; its value increases by 5% each year. Determine
    (a) an algebraic model for the coin's value over time
    (b) the coin's value ten years after it was bought
    (c) the coin's value 28 months after it was bought

33. What is the future value of the annuity in which you invest $360 per month earning an interest of 7.2%/a compounded monthly for 7 years?

34. Fatima takes out a mortgage of $200 000 amortized over 25 years. The bank offers a 5.25% interest rate compounded semi-annually for a 10-year term.
    a) Calculate the equivalent interest rate.
    b) Determine the monthly payment.
    c) Determine the total amount paid in 10 years.

35. Determine the values in the 5^{th} row of Pascal's triangle.

Answers:

1. **(a)** $D = \{x \mid x \le 1.5, x \in \mathbf{R}\}$, $R = \{y \mid y \in \mathbf{R}\}$, not a function, fails the vertical line test
   $D = \{x \mid x \in \mathbf{R}\}$, $R = \{y \mid y > 0, y \in \mathbf{R}\}$, function, passes the vertical line test
   $D = \{x \mid x \ge 5, x \in \mathbf{R}\}$, $R = \{y \mid y \ge 0, y \in \mathbf{R}\}$, function, every x value corresponds to a unique y value
   $D = \{x \mid x \in \mathbf{R}\}$, $R = \{y \mid y \ge 4, y \in \mathbf{R}\}$, function, every x value corresponds to a unique y value

2. a) 19 b) $\frac{38}{9}$ c) 5 d) $8x^2 - 6x + 5$ e) $a = 1$ or 0.5

3. Example: d) $y = -2\sqrt{5-10x}+1$ write as: $y = -2\sqrt{-10(x-0.5)}+1$
   - Reflection on the x-axis, vertical stretch, factor is 2
   - Reflection on the y-axis, horizontal compression, factor is $\frac{1}{10}$
   - Horizontal translation right 0.5 and vertical translation up 1.

*Factor the "k"-value!*i.e. a) $y = 3f[-4(x-2)]-5$ e) $y = \frac{2}{3}\sin\left[\frac{1}{4}(x+8)\right]-6$

4. **(b)** g^{-1} is $y = \pm\sqrt{\frac{2-x}{3}+1}+1$, **(c)** $D(g(x)) = \{x \mid x \ge 1, x \in \mathbf{R}\}$ or $D(g(x)) = \{x \mid x \le 1, x \in \mathbf{R}\}$
   (d) no, graph of g in that region fails the horizontal line test

5. a) $f(x) = 0.6x^2 - 3.6x + 0.4$ b) $f(x) = -2x^2 + 8x - 4$ c) answers will vary, same zeros or same vertex

6. a) $(-1, -9)$ and $\left(\frac{4}{3}, \frac{8}{3}\right)$ b) $y = -4x + k$. If $k = -1$, one sol^{n}, if $k > -1$, two sol^{ns} and if $k < -1$, no sol^{ns}.

7. a) $9\sqrt{3}$ b) $4\sqrt{6}$ c) $9\sqrt{5}$ d) $2\sqrt{3}$ e) $\frac{5-4\sqrt{2}}{7}$ f) $17-4\sqrt{15}$ g) 7

8. **(a)** $\frac{5x(x-1)}{(x-5)(x+1)}, x \ne 5, -1$ **(b)** $-\frac{4x}{5y}, x \ne 0, y \ne 0$ **(c)** $\frac{(2m+5)(m-3)^2}{m^2-m+9}, m \ne -2,$
   (d) $\frac{y(x+y)}{(x-4)(x-y)}, x \ne 0, 4, y; y \ne 0$ **(e)** $0, x \ne 0, y \ne 0$ **(f)** $\frac{-3x-1}{(x+3)(x-5)(x+2)}, x \ne -3, 5, -2$

9. a) $N(t) = 45000000\left(\frac{1}{3}\right)^t$ b) $y = -2^x + 3$ c) $y = 2(3^{-x}) + 2$ d) $y = -3(4^{x-5}) + 3$

10. No, it would take exactly 11 years.

11. a) 32 000 b) 2 048 000 c) $500\left(2^{2^n}\right)$

12. $x = -1$

13. a) –1 b) 16 c) 5^{m+1} d) $\frac{1}{16a^2b^6c^2}$ e) 1 f) $\frac{1}{8}$ g) $-\frac{1}{125}$ h) $a^{\frac{2}{3}}$ i) $\frac{1}{2}$ j) $\frac{1}{2}x^{\frac{1}{2}}$ k) $x^{\frac{3}{5}}$

14. **(a)** $\frac{142}{9}$ **(b)** $\frac{9x^2}{16y^2}$ **(c)** $\frac{2^{2n}x^{3m}}{2^m x^n}$

15. a) 1.2062 b) –1.7434 c) –0.8391

16. a) 67° or 293° b) 190° or 350° c) 115° or 295°

17. a) $\frac{2}{\sqrt{3}}$ b) $-\frac{2}{\sqrt{3}}$ c) –1

18. a) $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ b) $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ c) $\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ d) $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$ e) (0,–1)

19. a) 37°, 323°; b) 341°, 200°
20. Hint: expand out, do not re-write in terms of sin x and cos x.
21. a) hint: start with common denominators on the RHS
    b) hint: start with the LHS and use Pythagorean identities
22. a) one solution b) two solutions
23. $\angle N = 67°, \angle M = 75°, m = 47$ cm
24. 509 km
25. amplitude = 10, period = 4; answers will vary for the equation
26. 5, 3, 22, 94, 464, 2044
27. a) $d(t) \doteq 3.25\cos[0.26(t-0.1)]+14.75$, **(c)** 5:48 a.m and 6:12 p.m; 10 a.m and 2 p.m
28. **(a)** $t_n = -13 + 9n$, 212, 1630, **(b)** $t_n = 3(1.1)^{n-1}$, 29.55, 171.82
29. **(a)** 14 **(b)** 37
30. check your answer with a graphing program or graphing calculator
31. **(a)** \$18 768.34 **(b)** \$7472.58
32. **(a)** $t_n = \$1200(1.05)^n$ **(b)** \$1954.67 **(c)** \$1344.69
33. \$39 170.30
34. a) 0.43279 % b) \$1191.84 c) \$143 020.80
35. 1 4 6 4 1