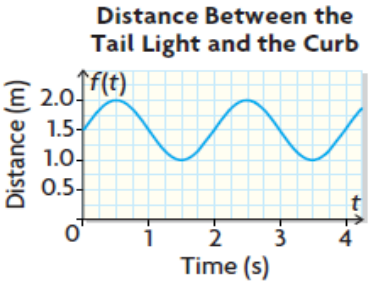
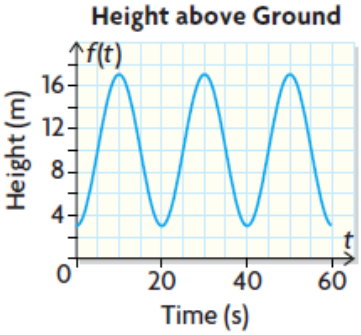
1. The load on a trailer has shifted, causing the rear end of the trailer to swing left and right. The distance from one of the tail lights on the trailer to the curb varies sinusoidally with time. The graph models this behaviour.



   a) What is the equation of the axis of the function, and what does it represent in this situation?
   b) What is the amplitude of the function, and what does it represent in this situation?
   c) What is the period of the function, and what does it represent in this situation?
   d) Determine the equation and the range of the sinusoidal function.
   e) What are the domain and range of the function in terms of the situation?
   f) How far is the tail light from the curb at t = 3 s?

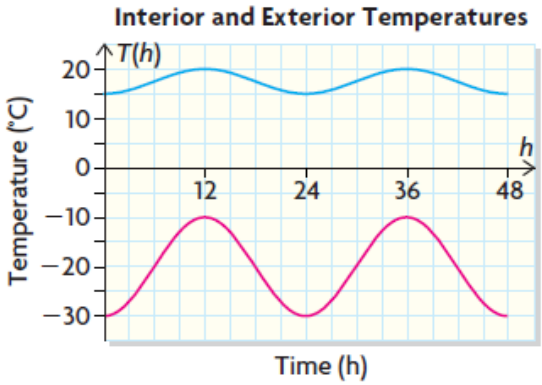
2. Don Quixote, a fictional character in a Spanish novel, attacked windmills because he thought they were giants. At one point, he got snagged by one of the blades and was hoisted into the air. The graph shows his height above ground in terms of time.



   a) What is the equation of the axis of the function, and what does it represent in this situation?
   b) What is the amplitude of the function, and what does it represent in this situation?
   c) What is the period of the function, and what does it represent in this situation?
   d) If Don Quixote remains snagged for seven complete cycles, determine the domain and range of the function.
   e) Determine the equation of the sinusoidal function.
   f) If the wind speed decreased, how would that affect the graph of the sinusoidal function?

3. Chantelle is swinging back and forth on a trapeze. Her distance from a vertical support beam in terms of time can be modelled by a sinusoidal function. At $1\,s$, she is the maximum distance from the beam, $12\,m$. At $3\,s$, she is the minimum distance from the beam, $4\,m$. Determine an equation of a sinusoidal function that describes Chantelle's distance from the vertical beam in relation to time.

4. The interior and exterior temperatures of an igloo were recorded over a $48\,h$ period. The data were collected and plotted, and two curves were drawn through the appropriate points.



   a) How are these curves similar? Explain how each of them might be related to this situation.
   b) Describe the domain and range of each curve.
   c) Assuming that the curves can be represented by sinusoidal functions, determine the equation of each function.

5. Skyscrapers sway in high-wind conditions. In one case, at $t=2\,s$, the top floor of a building swayed 30 cm to the left $(-30\,cm)$ and at $t=12\,s$, the top floor swayed 30 cm to the right $(+30\,cm)$ of its starting position.
   a) What is the equation of a sinusoidal function that describes the motion of the building in terms of time?

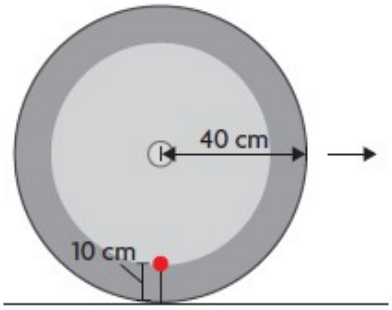
b) Dampers, in the forms of large tanks of water, are often added to the top floors of skyscrapers to reduce the severity of the sways. If a damper is added to this building, it will reduce the sway (not the period) by 70%. What is the equation of the new function that describes the motion of the building in terms of time?

6. Milton is floating in an inner tube in a wave pool. He is $1.5\,m$ from the bottom of the pool when he is at the trough (bottom) of a wave. A stopwatch starts timing at this point. In $1.25\,s$, he is on the crest (top) of the wave, $2.1\,m$ from the bottom of the pool.
   a) Determine the equation of the function that expresses Milton's distance from the bottom of the pool in terms of time.
   b) What is the amplitude of the function, and what does it represent in this situation?
   c) How far above the bottom of the pool is Milton at $t=4\,s$?
   d) If data are collected for only 40 s, how many complete cycles of the sinusoidal function will there be?
   e) If the period of the function changes to 3 s, what is the equation of this new function?

7. An oscilloscope hooked up to an alternating current (AC) circuit shows a sine curve. The device records the current in amperes (A) on the vertical axis and the time in seconds on the horizontal axis. At $t=0\,s$, the current reads its first maximum value of $4.5\,A$. At $t=\frac{1}{120}\,s$, the current reads its first minimum value of $-4.5\,A$. Determine the equation of the function that expresses the current in terms of time.

8. Candice is holding onto the end of a spring that is attached to a lead ball. As she moves her hand slightly up and down, the ball moves up and down. With a little concentration, she can repeatedly get the ball to reach a maximum height of $20\,cm$ and a minimum height of $4\,cm$ from the top of a surface. The first maximum height occurs at $0.2\,s$, and the first minimum height occurs at $0.6\,s$.
   a) Determine the equation of the sinusoidal function that represents the height of the lead ball in terms of time.
   b) Determine the domain and range of the function.
   c) What is the equation of the axis, and what does it represent in this situation?
   d) What is the height of the lead ball at 1.3 s?

9. A paintball is shot at a wheel of radius 40 cm. The paintball leaves a circular mark $10\,cm$ from the outer edge of the wheel. As the wheel rolls, the mark moves in a circular motion.
   a) Assuming that the paintball mark starts at its lowest point, determine the equation of the sinusoidal function that describes the height of the mark in terms of the distance the wheel travels.
   b) If the wheel completes five revolutions before it stops, determine the domain and range of the sinusoidal function.
   c) What is the height of the mark when the wheel has travelled $120\,cm$ from its initial position?



10. The population of rabbits, $R(t)$, and the population of foxes, $F(t)$, in a given region are modelled by the functions $R(t)=10000+5000\cos(15^\circ t)$ and $F(t)=1000+500\sin(15^\circ t)$, where t is the time in months. Referring to each graph, explain how the number of rabbits and the number of foxes are related.

Hints:

Question # 1:

- the left/right movement of the trailer is represented vertically (y-values) on the graph

Question # 2:

- Don Quixote is snagged by the windmill at the bottom of its rotation, which is a minimum on the graph

Question # 3:

- her horizontal distance (swinging back-and-forth) will be vertical on the graph (y-values)
- the x-axis of the graph (if you choose to draw one, which is recommended) is time
- the time between the maximum and minimum distance on the graph is **half** of a period

Question # 4:

- for similarities, compare key features (amplitude, period, axis, and phase shift)
- remember that you can choose to model each curve using sine or cosine, and your decision will influence the phase shift (starting point) and any vertical reflections, but amplitude and period will be unchanged

Question # 5:

- the question refers to horizontal motion, but this is represented as vertical in any graph you choose to draw, as well as the equation (the y-values)
- you will need to think back to a previous unit, exponential functions, to come up with the new equation (which is no longer a periodic function!)

Question # 6:

- the stopwatch starts when he is at the bottom of a wave, so t=0 is a minimum, and t=1.25 is a max

Question # 7:

- an oscilloscope is a device for measuring electrical signals, particularly those that change regularly (periodic)
- you may want to change the horizontal (time) scale to a decimal, although it is great if you can work through this using fractions

Question # 8:

- you know the times of the first max and first min, which is half of a period
- a sketch of the graph would probably help

Question # 9:

- the radius of the wheel is 40cm, but the paintball isn't on the outer edge, it is 10cm in from the edge, so 30cm from the middle (this is the radius of the circle that the paintball makes)
- this question is actually very similar to our first examples using the Ferris wheel, with a starting point (minimum) above the ground (zero)
- for (c), first consider how many rotations (or revolutions) of the wheel are required to travel 120cm
    - consider the circumference of the wheel using a radius of 40cm
    - once you know the number of revolutions, you can calculate the number of periods
    - finally, use the number of periods (expect a decimal or fraction) and sub into your equation for the paintball to find the height of the mark

Question #10:

- for questions dealing with similarities, compare key features such as amplitude, period, axis of the curve, and phase shift (i.e., starting point)
- you may also want to compare when the maximum and minimum values occur, and comment on any pattern you may see
    - this is a predator-prey relationship, so even without the equations or graphs, you should be able to come up with a reasonable hypothesis on how these populations compare to each other