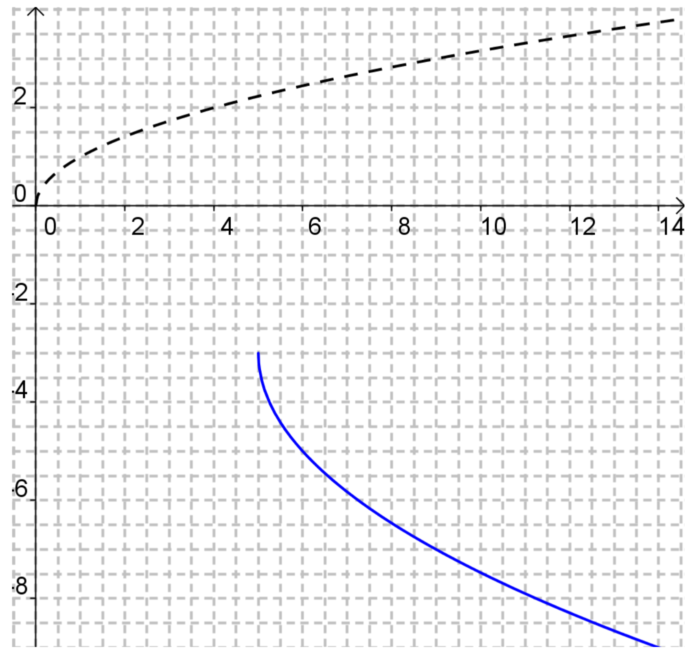
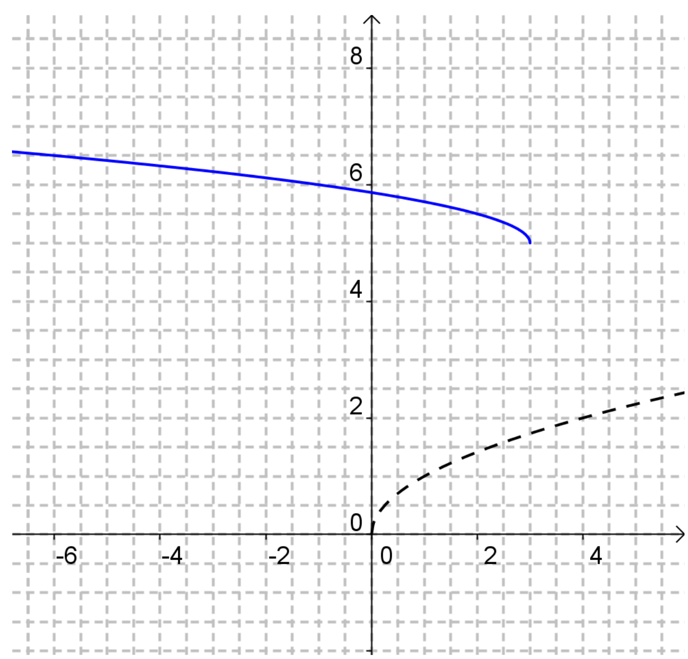
Ex.1 The diagram below shows a transformed radical function as well as the parent function. Determine the transformations involved and express the entire transformation using function notation.

- $(0,0) \rightarrow (5,-3)$: $p=5$ and $q=-3$
- vertical reflection: $a<0$
- for radical function, choose to look only at k, so $|a|=1$ and $a=-1$
- looks like a horizontal compression, so try to find where the point $(4,2)$ has moved on the new graph
    - choose $(4,2)$ because it's a compression, so start with a bigger x-value (4) and see how much smaller it gets
    - the y-value (2) will be the same
    - a step of 4 in the x becomes a step of 1
    - $|k|=4$ (compression by 4)
- $y=-f[4(x-5)]-3$



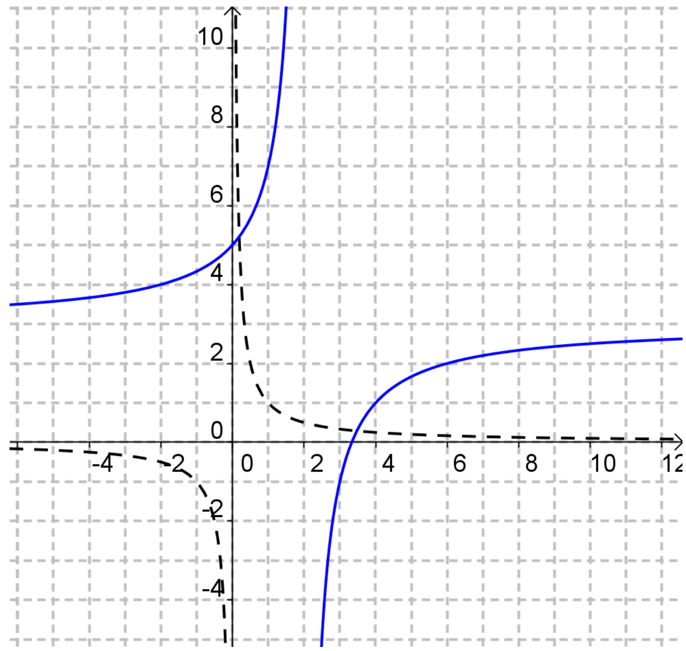
Ex.2 The diagram below shows a transformed radical function as well as the parent function. Determine the transformations involved and express the entire transformation using function notation.

- $(0,0) \rightarrow (3,5)$: $p=3$ and $q=5$
- horizontal reflection: $k<0$
- for radical function, choose to look only at k, so $|a|=1$ and $a=-1$
- looks like a horizontal stretch, so try to find where the point $(1,1)$ has moved on the new graph
    - choose $(1,1)$ because it's a stretch, so start with a smaller x-value (1) and see how much bigger it gets
    - the y-value (1) will be the same
    - a step of 1 in the x becomes a step of 4
    - $|k|=\frac{1}{4}$ (stretch by 4)
- $y=f\left[-\frac{1}{4}(x-3)\right]+5$



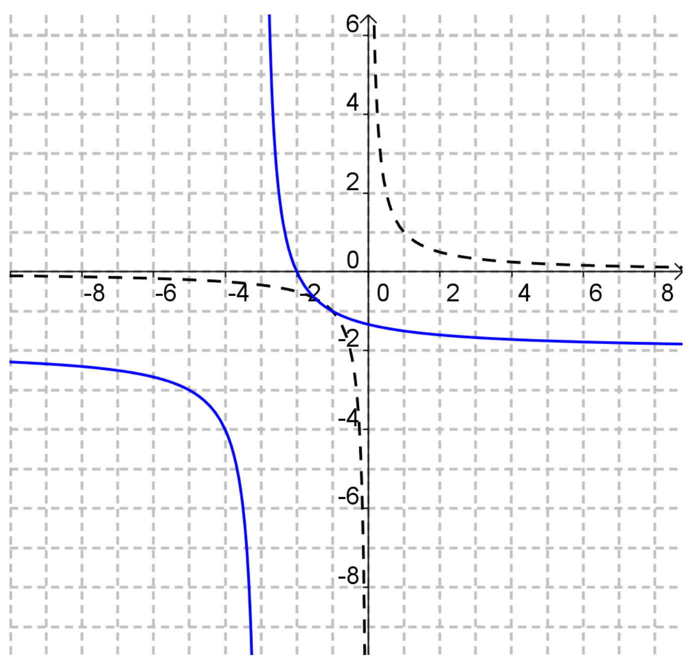
Ex.3 The diagram below shows a transformed reciprocal function as well as the parent function. Determine the transformations involved and express the entire transformation using function notation.

- asymptotes: $x=0 \rightarrow x=2$ and $y=0 \rightarrow y=3$
  $p=2$ and $q=3$
- for reciprocal, either reflection is fine
  - choose vertical reflection: $a<0$
- for reciprocal, either scaling is fine
  - choose vertical scaling, so $k=1$
- consider vertical distance from asymptote to point $(1,1)$, so $d_1=1$
- now consider vertical distance from new asymptote ($y=3$) to new point $(1,-1)$
  - remember to measure distance from asymptote, which is $d_2=4$
- vertical stretch by 4, $|a|=4$, $a=-4$
- $y=-4f(x-2)+3$



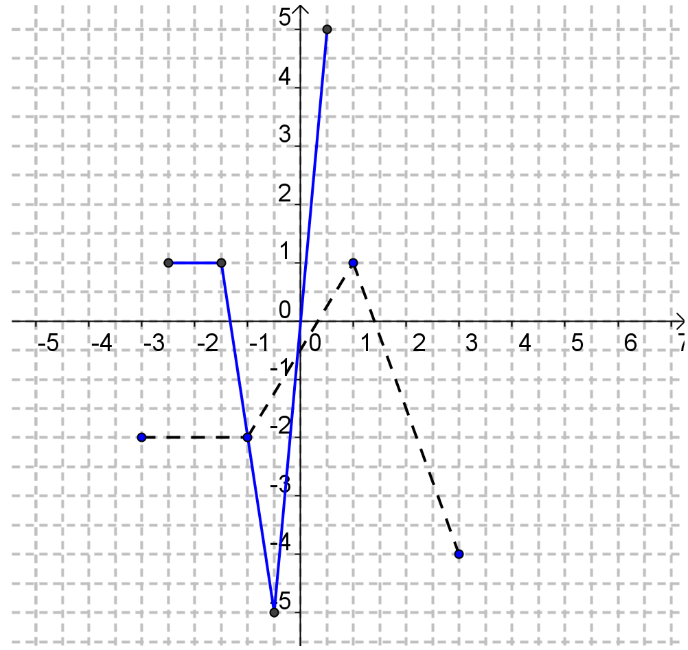
Ex.4 The diagram below shows a transformed reciprocal function as well as the parent function. Determine the transformations involved and express the entire transformation using function notation.

- asymptotes: $x=0 \rightarrow x=-3$ and $y=0 \rightarrow y=-2$
  $p=-3$ and $q=-2$
- no reflection
- for reciprocal, either scaling is fine
  - choose vertical scaling, so $k=1$
- consider vertical distance from asymptote to point $(1,1)$, so $d_1=1$
- now consider vertical distance from new asymptote ($y=-2$) to new point $(-2,0)$
  - remember to measure distance from asymptote, which is $d_2=2$
- vertical stretch by 2, $|a|=2$, $a=2$
- $y=2f(x+3)-2$



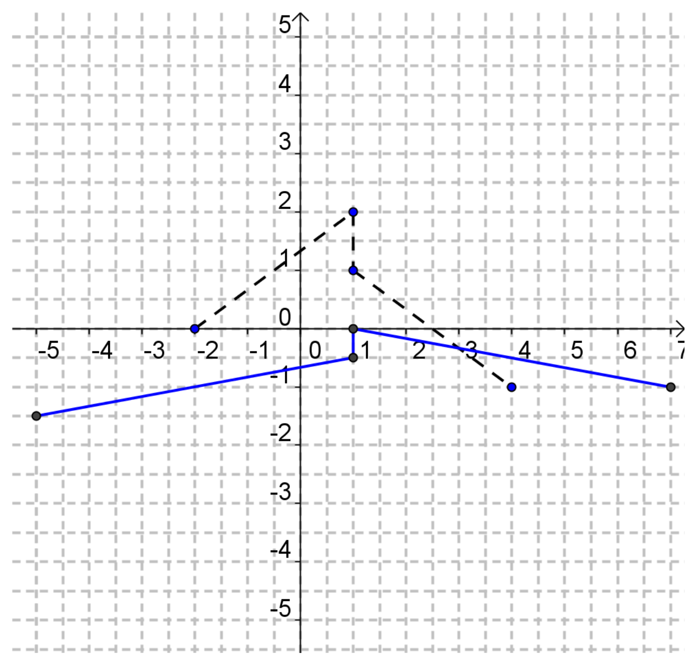
Ex.5 The diagram below shows a transformed piecewise function as well as the parent function. Determine the transformations involved and express the entire transformation using function notation.

- no well-defined zero-values to work with
- vertical reflection
- compare full-widths and full-heights
- $w_1 = 6$, $w_2 = 3$
  - compress by 2, $|k| = 2$, $k = 2$
- $h_1 = 5$, $h_2 = 10$
  - stretch by 2, $|a| = 2$, $a = -2$
- use $(x, y) \rightarrow \left(\frac{x}{k} + p, a\,y + q\right)$ to find p, q
  - $(1,1) \rightarrow (-0.5, -5)$
  - $\frac{1}{2} + p = -0.5$, $p = -1$
  - $-2(1) + q = -5$, $q = -3$
- $y = -2\,f\left[2(x+1)\right] - 3$



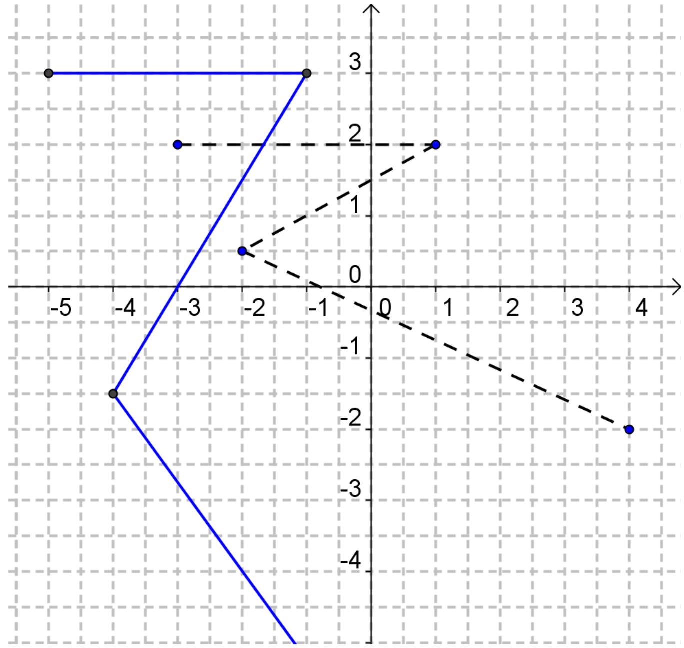
Ex.6 The diagram below shows a transformed piecewise function as well as the parent function. Determine the transformations involved and express the entire transformation using function notation.

- $(-2, 0) \rightarrow (7, -1)$ so $q = -1$
- horizontal reflection, $k < 0$, $a > 0$
- compare full-widths and full-heights
- $w_1 = 6$, $w_2 = 12$
  - h. stretch by 2, $|k| = \frac{1}{2}$, $k = -\frac{1}{2}$
- $h_1 = 3$, $h_2 = 1.5$
  - compress by 2, $|a| = \frac{1}{2}$, $a = \frac{1}{2}$
- use $(x, y) \rightarrow \left(\frac{x}{k} + p, a\,y + q\right)$ to find p
  - $(1,1) \rightarrow (1, -0.5)$
    $-2x + p = 1$
  - $-2(1) + p = 1$
    $p = 3$
- $y = \frac{1}{2} f\left[-\frac{1}{2}(x-3)\right] - 1$



Ex.7 The diagram below shows a transformed piecewise function as well as the parent function. Determine the transformations involved and express the entire transformation using function notation.

- no zero-values to work with
- no reflections, $k>0$, $a>0$
- one point missing, so compare widths and heights between key points
- top 2 points: $w_1=4$, $w_2=4$, $k=1$
- 2nd & 3rd points: $h_1=1.5$, $h_2=4.5$
  - v.stretch by 3, $|a|=3$, $a=3$
- use $(x,y) \rightarrow \left(\frac{x}{k}+p, ay+q\right)$ to find p, q
  - $(1,2) \rightarrow (-1,3)$
    $$\begin{aligned} x+p &= -1 & 3y+q &= 3 \\ (1)+p &= -1 & 3(2)+q &= 3 \\ p &= -2 & q &= -3 \end{aligned}$$
- $y=3f(x+2)-3$



Ex.8 The diagram below shows a transformed piecewise function as well as the parent function. Determine the transformations involved and express the entire transformation using function notation.

- $(0,4) \rightarrow (2,3)$: right 2, $p=2$
- $(-6,0) \rightarrow (4,5)$: up 5, $q=5$
- difficult to see (sorry), but there is actually a vertical and horizontal reflection: $k<0$, $a<0$
- compare width: $w_1=9$, $w_2=3$
  - h.compress by 3: $|k|=3$, $k=-3$
- compare height: $h_1=7$, $h_2=3.5$
  - v.compress by 2: $|a|=2$, $a=-2$
- $y=-2f[-3(x-2)]+5$