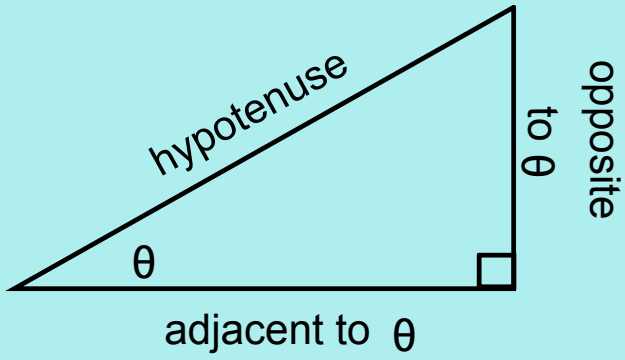
Recall:

For any angle of interest (θ), there are three (3) primary trigonometric ratios.

$$\text{sine of } \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

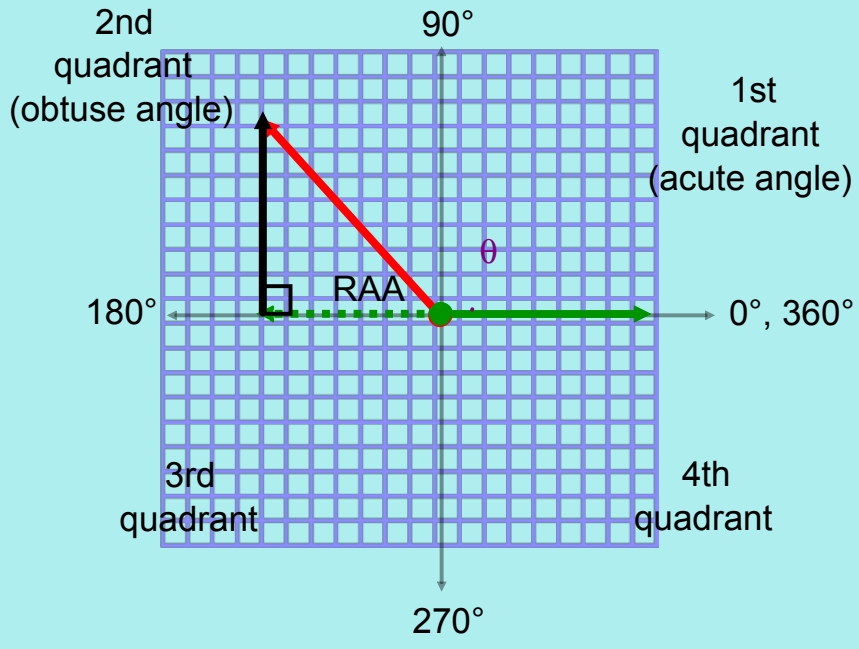
$$\text{cosine of } \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\text{tangent of } \theta = \frac{\text{opposite}}{\text{adjacent}}$$



S o h C a h T o a

To work with angles greater than 90°, we form a right-triangle using the terminal arm and the related acute angle.



Trigonometry of Any Angle: The CAST Rule

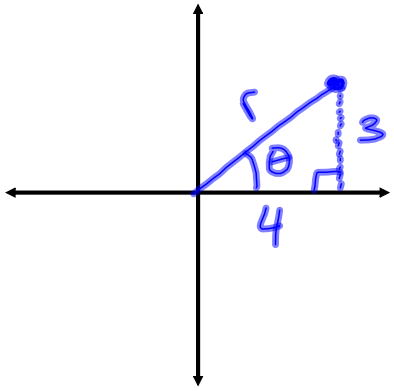
Nov. 19/2013

Any angle in standard position has a related acute angle (RA, RAA).
A right-triangle can always be drawn using this RAA.

Therefore any angle can be associated with the primary trig ratios.

The quadrant will determine the sign of the ratio.

Ex.1 (a) Consider P(4, 3)



RAA = θ

$3^2 + 4^2 = r^2$

$25 = r^2$

$r = \pm 5$

$r = 5$

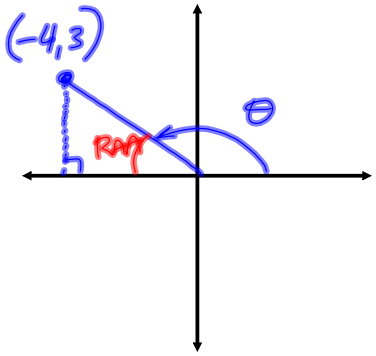
(distance is positive)

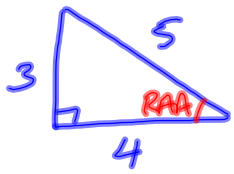
$\sin\theta = \frac{o}{h}$ $\cos\theta = \frac{a}{h}$ $\tan\theta = \frac{o}{a}$

$\sin\theta = \frac{3}{5}$ $\cos\theta = \frac{4}{5}$ $\tan\theta = \frac{3}{4}$

$\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{y}{x}$

Ex.1 (b) Consider P(-4, 3)





$\sin RAA = \frac{3}{5}$

$\cos RAA = \frac{4}{5}$

$\tan RAA = \frac{3}{4}$

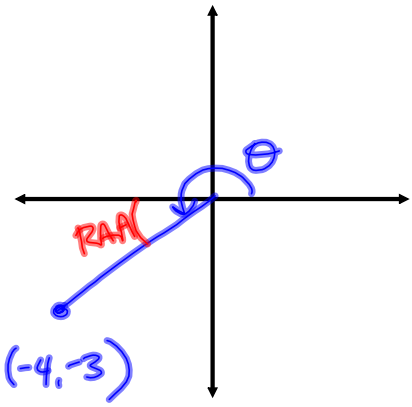
Using new definitions:

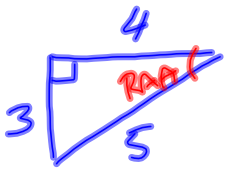
$\sin\theta = \frac{y}{r}$ $\quad \cos\theta = \frac{x}{r}$ $\quad \tan\theta = \frac{y}{x}$

$= \frac{3}{5}$ $\quad = \frac{-4}{5}$ $\quad = \frac{3}{-4}$

$= \sin RAA$ $\quad = -\cos RAA$ $\quad = -\tan RAA$

Ex.1 (c) Consider P(-4, -3)





$\sin RAA = \frac{3}{5}$

$\cos RAA = \frac{4}{5}$

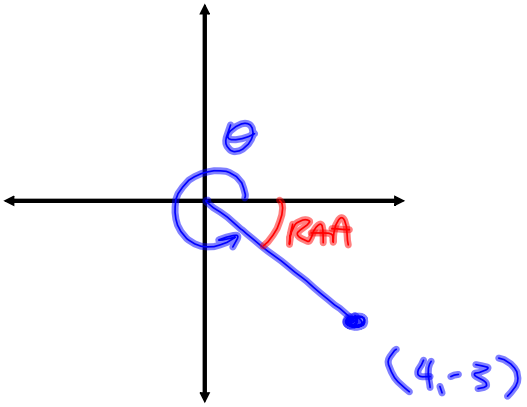
$\tan RAA = \frac{3}{4}$

$\sin\theta = \frac{y}{r}$ $\quad \cos\theta = \frac{x}{r}$ $\quad \tan\theta = \frac{y}{x}$

$= \frac{-3}{5}$ $\quad = \frac{-4}{5}$ $\quad = \frac{-3}{-4}$

$= -\sin RAA$ $\quad = -\cos RAA$ $\quad = \tan RAA$

Ex.1 (d) Consider P(4, -3)



$\sin\theta = \frac{y}{r} = \frac{-3}{5} = -\sin RAA$

$\cos\theta = \frac{x}{r} = \frac{4}{5} = \cos RAA$

$\tan\theta = \frac{y}{x} = \frac{-3}{4} = -\tan RAA$

The CAST rule allows us to quickly determine the sign of each trig ratio for any quadrant.

sin + cos - tan -	sin + cos + tan +
sin - cos - tan +	sin - cos + tan -

S	A
T	C

Ex.2 Predict the sign of each value (verify with calculator)

(a) tan 135°

negative

(b) cos 240°

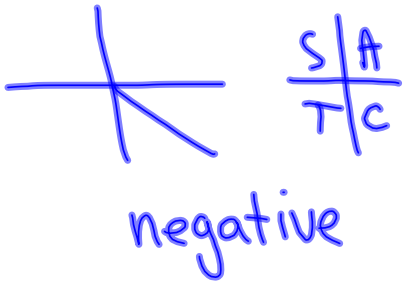
negative

(c) sin 430°

$= \sin 70°$ coterminal angles

positive

(d) tan(-30°)



negative

Ex.3 For $\tan\theta = -\frac{5}{24}$, where $0° \le \theta \le 360°$

(a) where (i.e., which quadrant) is θ?

tan θ is negative

θ is in Q2 or Q4

(b) What are the values of x, y, and r?

$\tan\theta = \frac{y}{x}$

$-\frac{5}{24} = \frac{y}{x}$

$y = 5$, $x = -24$ — Q2

$y = -5$, $x = 24$ — Q4

$r^2 = x^2 + y^2$

$r^2 = (-24)^2 + 5^2$

$r^2 = 576 + 25$

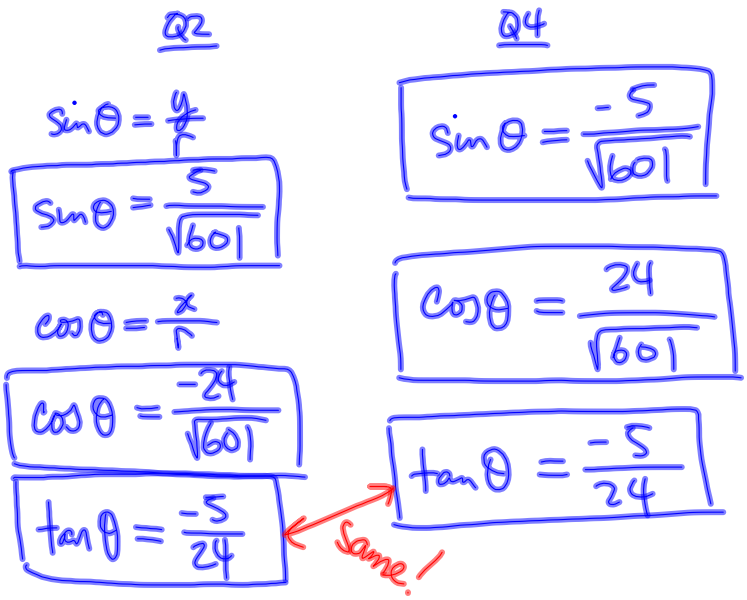
$r^2 = 601$

$r = \sqrt{601}$

(discard negative because r is a distance)

Ex.3 For $\tan\theta = -\frac{5}{24}$, where $0° \le \theta \le 360°$

(c) determine the trig ratios in each possible quadrant



Assigned Work:

~~WS # 1-4~~

p. 348 #1, 2, 6, 7
new

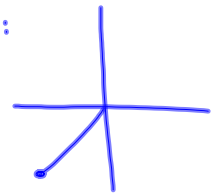
$2\pi \longrightarrow 360°$

7(f) $\sin\theta = -\frac{5}{6}$ $\quad 0° \le \theta < 360°$ (2π)

S | A
T | C

θ is in Q3 or Q4

Q3:



$x^2 + y^2 = r^2$

$x^2 + (-5)^2 = 6^2$

$x^2 + 25 = 36$

$x^2 = 11$

$x = \pm\sqrt{11}$

$\sin\theta = \frac{y}{r}$, $-\frac{5}{6} = \frac{y}{r}$ $\quad y = -5$, $r = 6$

$x = -\sqrt{11}$ in Q3

$x = \sqrt{11}$ in Q4

Ratios:

in Q3: $\sin\theta = \frac{-5}{6}$

$\cos\theta = \frac{-\sqrt{11}}{6}$

$\tan\theta = \frac{-5}{-\sqrt{11}} = \frac{5}{\sqrt{11}}$

in Q4:

$\sin\theta = -\frac{5}{6}$

$\cos\theta = \frac{\sqrt{11}}{6}$

$\tan\theta = -\frac{5}{\sqrt{11}}$