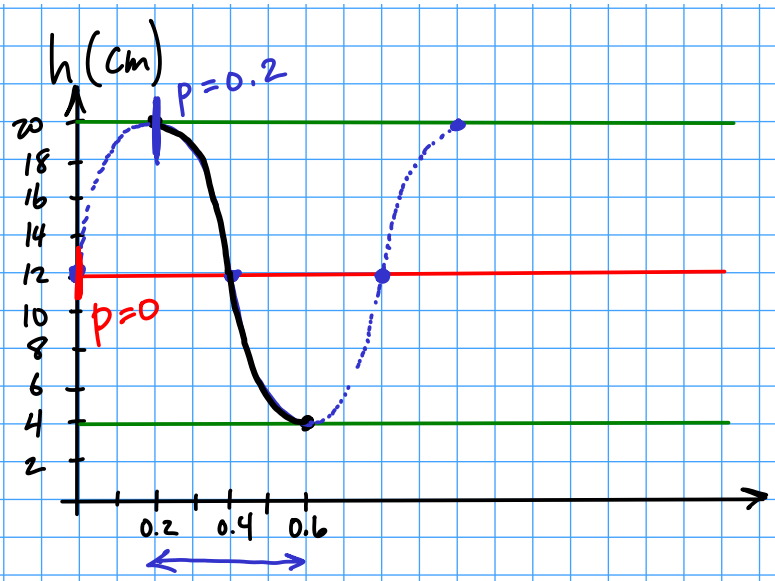
8. Candice is holding onto the end of a spring that is attached to a lead ball. As she moves her hand slightly up and down, the ball moves up and down. With a little concentration, she can repeatedly get the ball to reach a maximum height of $20\,cm$ and a minimum height of $4\,cm$ from the top of a surface. The first maximum height occurs at $0.2\,s$, and the first minimum height occurs at $0.6\,s$.
   a) Determine the equation of the sinusoidal function that represents the height of the lead ball in terms of time.
   b) Determine the domain and range of the function.
   c) What is the equation of the axis, and what does it represent in this situation?
   d) What is the height of the lead ball at 1.3 s?



① AofC: $y = \frac{\max + \min}{2}$

$= \frac{20+4}{2}$

$y = 12 \rightarrow q = 12$

② $|a| = \frac{20-4}{2}$

$= 8$

③ $\frac{1}{2}$ period $= 0.6 - 0.2$

$\frac{1}{2}T = 0.4$

$T = 0.8$

$k = \frac{360°}{T}$

$= \frac{360°}{0.8}$

$= 450°$

④ choose $p = 0.2$ (most obvious choice using given points)

$a = 8$ (regular cosine)

$h(t) = 8\cos[450°(t - 0.2)] + 12$

OR choose $p = 0$ (need to work backwards on graph to find another point)

$a = 8$ (regular sine)

$h(t) = 8\sin(450°t) + 12$

OR choose $p = 0.6$ (2nd point provided in question)
$a = -8$ (reflected cosine)

$$h(t) = -8\cos[450^\circ(t-0.6)] + 12$$

(b) $D = \{t \mid t \in \mathbb{R}, t \geq 0s\}$, which represents the time where she can keep repeating this motion consistently.

$R = \{h \mid h \in \mathbb{R}, 4cm \leq h \leq 20cm\}$, which represents the possible heights for the mass on the pendulum.

(c) Axis of the curve is $y = 12cm$, which represents the "rest position" of the mass on the spring (i.e., the height it would hang with no movement).

(d) At $t = 1.3s$, $h(1.3) = 8\cos[450^\circ(1.3-0.2)] + 12$
$= 8\cos[450^\circ(1.1)] + 12$
$\doteq 6.3$

∴ at 1.3 seconds, the mass is 6.3 cm above the surface.