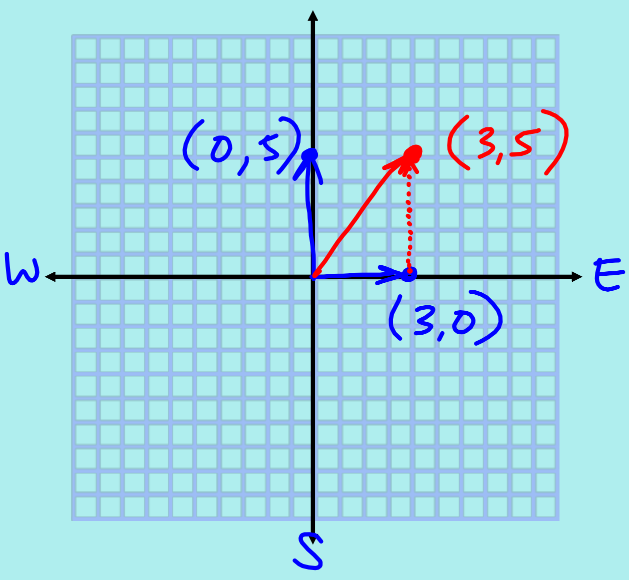
Vectors in R^2 and R^3

Apr. 25/2014.

Consider & draw the following vectors:

5 km [north]
3 km [east]



What are the coordinates of their end points?

5 km [N] → (0,5)
3 km [E] → (3,0)

What if you added them together? What is the resultant vector, and what are the coordinates of the end point?

P(3,5)

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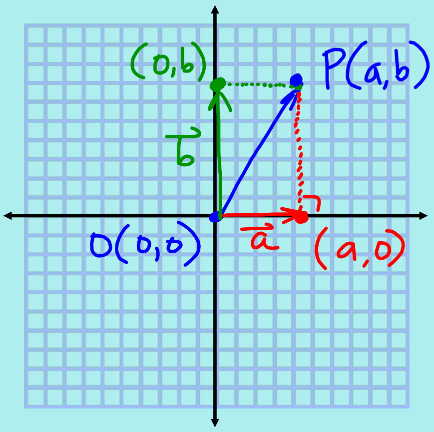
Consider a generic point,

$P(a, b)$

The vector from the origin (O) to P would be

$\overrightarrow{OP}$

Can we relate this vector to the x- and y-axes?



$\overrightarrow{OP} = \vec{a} + \vec{b}$ ← x and y component vectors

$= (a, b)$ ← vector from O(0,0) to P(a,b)

$= [a, b]$ ← alternate representation

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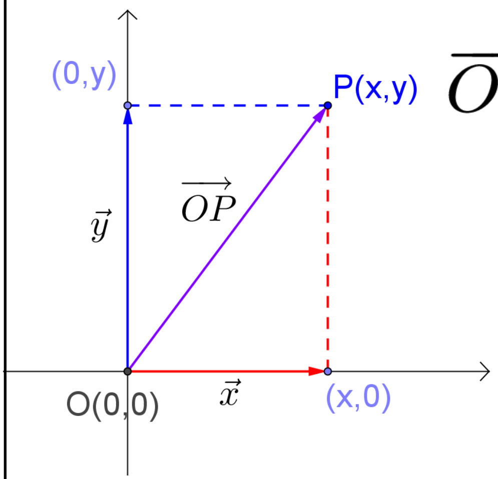
Vectors in R^2 and R^3

Apr. 25/2014

The x-y plane, R^2, spans two dimensions (e.g., length & width).

Any point P(x,y) can be located in terms of a vector from the origin O(0,0) to the point P(x,y). This is the position vector of the point P.



$$\overrightarrow{OP} = \vec{x} + \vec{y} = (x, y)$$

The magnitude, or length, of the vector is given by

$$\left|\overrightarrow{OP}\right| = \sqrt{x^2 + y^2}$$

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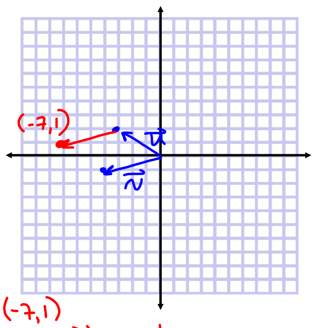
Ex.1 Given vectors:

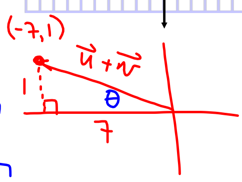
$$\vec{u} = (-3, 2)$$
$$\vec{v} = (-4, -1)$$

(a) sketch $\vec{u}$, $\vec{v}$, and $\vec{u} + \vec{v}$



(b) find the magnitude and direction of $\vec{u} + \vec{v}$ showing triangle addition.

(c) find $\vec{u} + \vec{v}$ algebraically



(b) $|\vec{u}+\vec{v}| = \sqrt{x^2 + y^2}$
$= \sqrt{(-7)^2 + (1)^2}$
$= \sqrt{50}$
$= 5\sqrt{2}$

$\tan\theta = \frac{1}{7}$
$\theta = \tan^{-1}\left(\frac{1}{7}\right)$
$\theta \doteq 8.1^\circ$

$\therefore$ $\vec{u}+\vec{v}$ is $5\sqrt{2}$ in length, at 8.1° above $-x$ axis.

(c) $\vec{u}+\vec{v} = (-3, 2) + (-4, -1)$
$= (-7, 1)$

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Vectors in 3-Dimensional Space

What is 3-D?

3 dimensional means there are 3 different planes

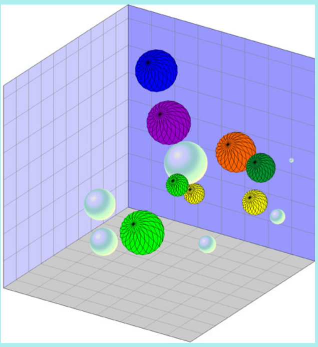
How do we graph 3-D?

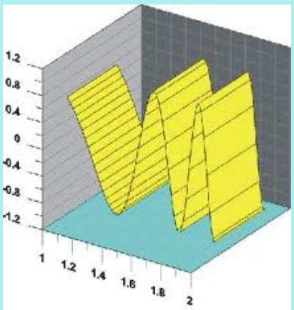
In order to graph in 3 dimensions we need a third axis (z axis)

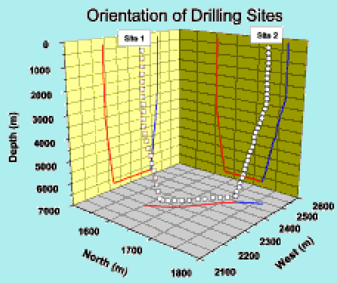
What are the co-ordinates of a 3-dimensional graph?

The co-ordinates of a 3-dimensional graph would be written (x,y,z)

What does a three dimensional grid look like?







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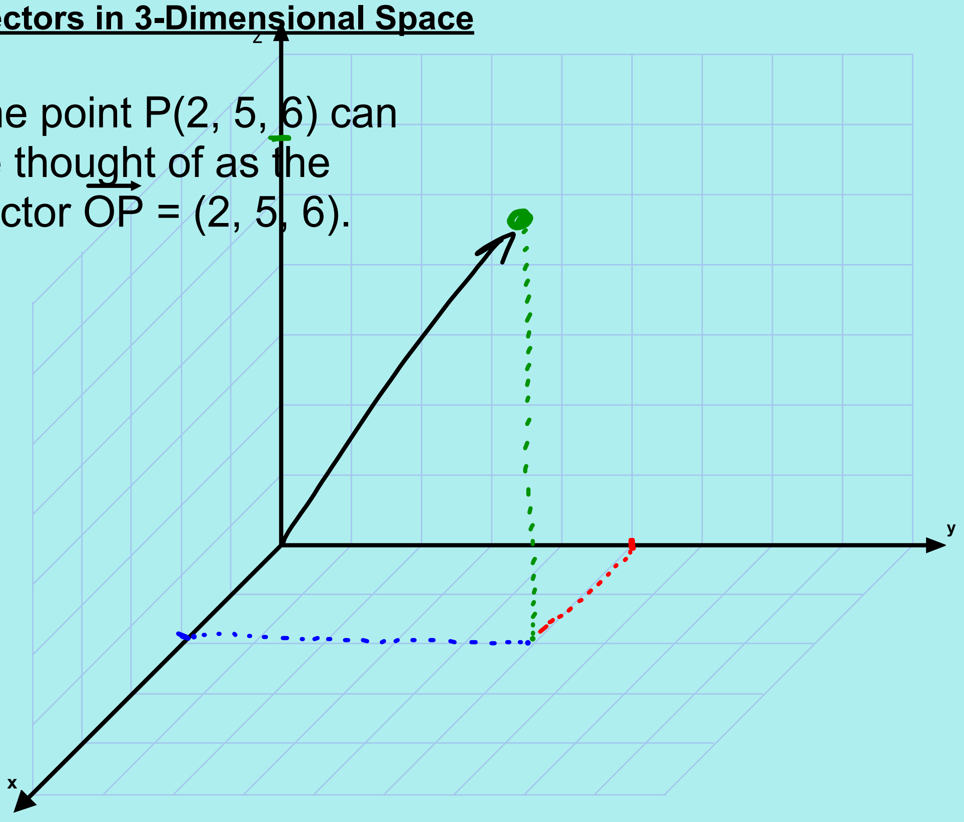
Vectors in 3-Dimensional Space

The point P(2, 5, 6) can be thought of as the vector $\overrightarrow{OP} = (2, 5, 6)$.



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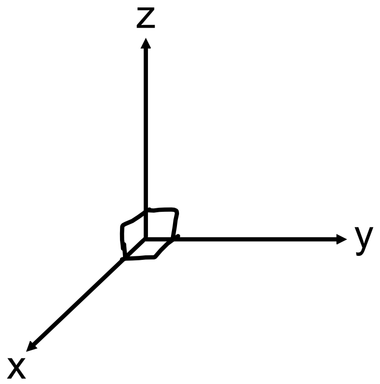
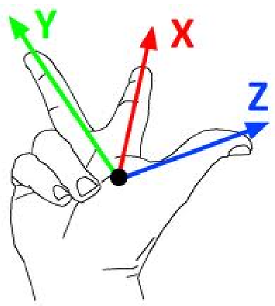
In three dimensions, we add the z-axis, forming R^3. The three axes (x, y, and z) form a perpendicular, right-handed system.





Any point, P(x,y,z), can be described by the position vector from the origin O(0,0,0) to the point P(x,y,z).

$$\overrightarrow{OP} = (x, y, z) \qquad \left|\overrightarrow{OP}\right| = \sqrt{x^2 + y^2 + z^2}$$

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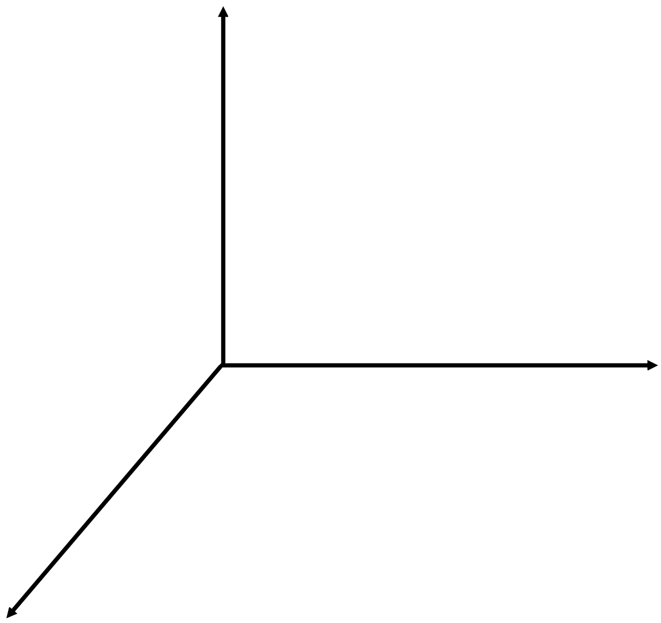
Ex2: Draw these vectors within a corresponding rectangular prism (if needed) in $\mathbf{R}^3$.

$\vec{u} = (2, 3, 4)$

$\vec{v} = (3, -4, 5)$

$\vec{w} = (0, -3, 0)$



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Assigned work:

p. 316 # 1, 4, 5, 6, 7ab, 8, 12a, 15

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