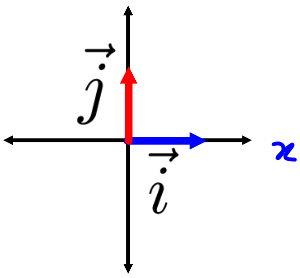
Operations with Algebraic Vectors in R^2

Apr. 28/2014

Recall: A unit vector is a vector of length one (1).



For unit vectors along the x- and y-axes, we use:

$$\vec{i} = (1, 0)$$
$$\vec{j} = (0, 1)$$

These are the standard basis vectors for R^2, meaning that any vector in R^2 can be expressed in terms of $\vec{i}$ and $\vec{j}$.

Any point, P(a,b), can be represented as an algebraic vector expressed in terms of the unit vectors:

$$\overrightarrow{OP} = (a, b) = a\vec{i} + b\vec{j}$$

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Adding Vectors

$\overrightarrow{OA} + \overrightarrow{OD}$

$= (a, b) + (c, d)$

$= 1(a\vec{i} + b\vec{j}) + 1(c\vec{i} + d\vec{j})$

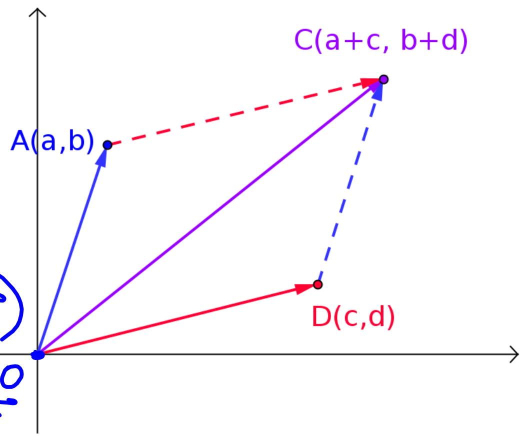
$= a\vec{i} + b\vec{j} + c\vec{i} + d\vec{j}$

$= a\vec{i} + c\vec{i} + b\vec{j} + d\vec{j}$

$= (a+c)\vec{i} + (b+d)\vec{j}$

$= (a+c, b+d)$

$= \overrightarrow{OC}$



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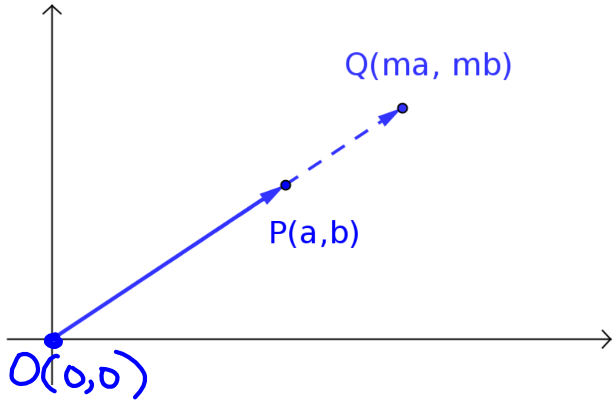
Scalar Multiplication of Vectors

Given $P(a, b)$

$$m\overrightarrow{OP}$$
$$= m(a, b)$$
$$= m(a\vec{i} + b\vec{j})$$
$$= ma\vec{i} + mb\vec{j}$$
$$= (ma, mb)$$
$$= \overrightarrow{OQ}$$



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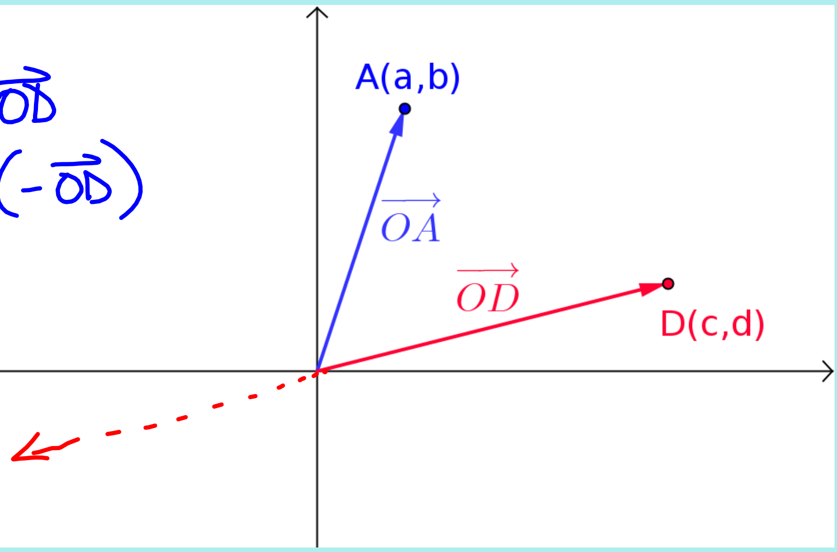
Difference of Vectors

$$\overrightarrow{OA} - \overrightarrow{OD}$$
$$= \overrightarrow{OA} + (-\overrightarrow{OD})$$



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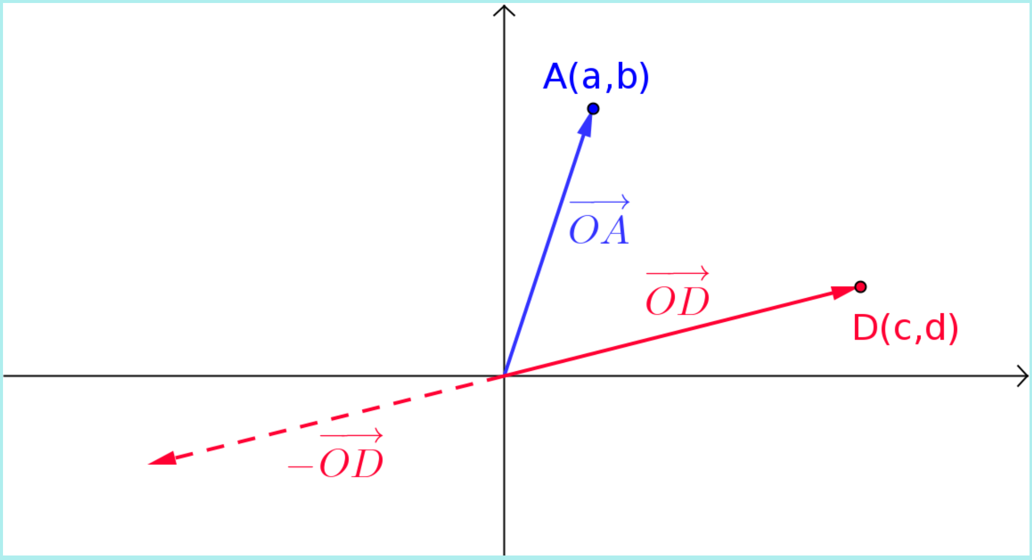
Difference of Vectors



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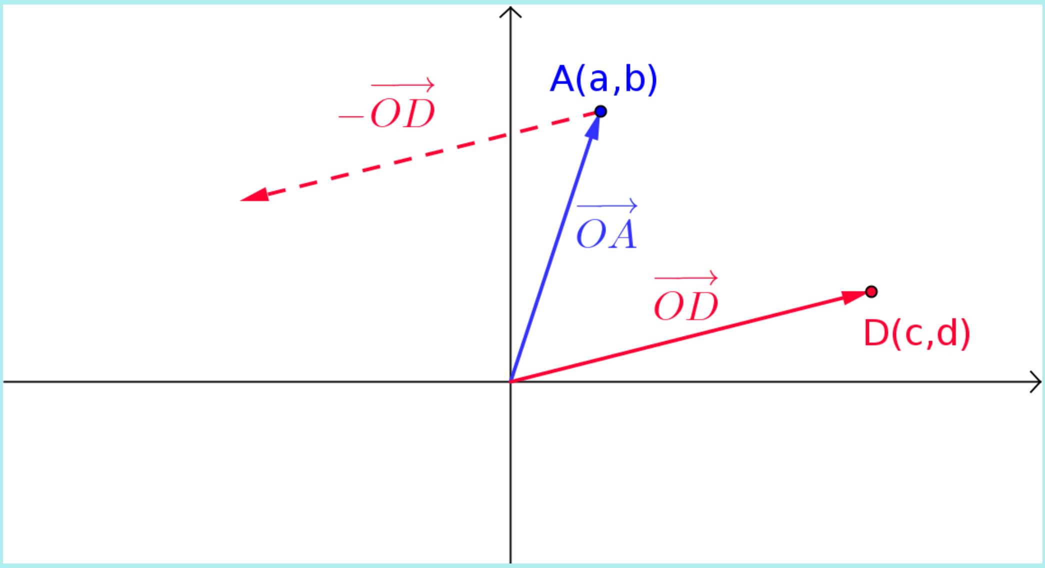

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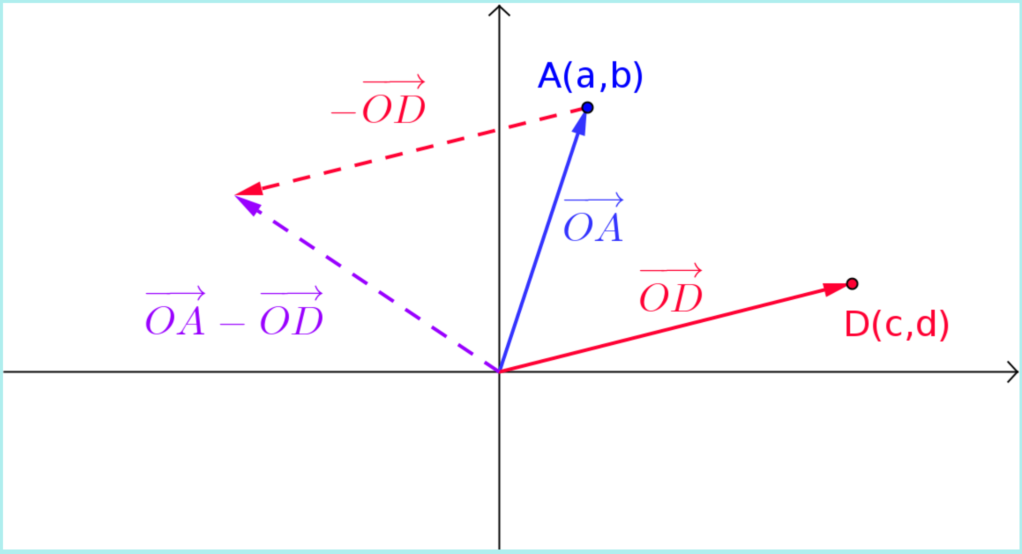
Difference of Vectors



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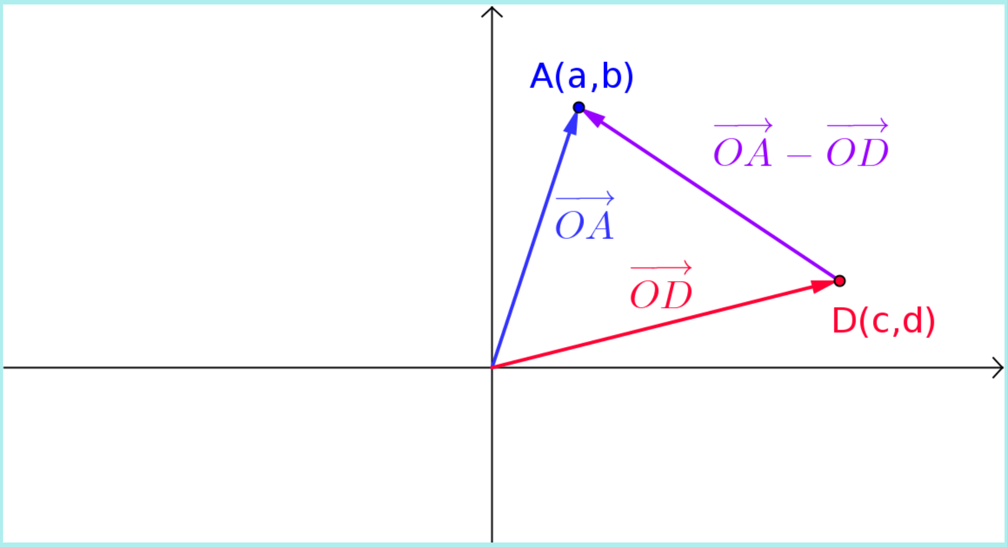
Difference of Vectors



$\overrightarrow{DA} = \overrightarrow{OA} - \overrightarrow{OD}$

$\overrightarrow{AD} = \overrightarrow{OD} - \overrightarrow{OA}$

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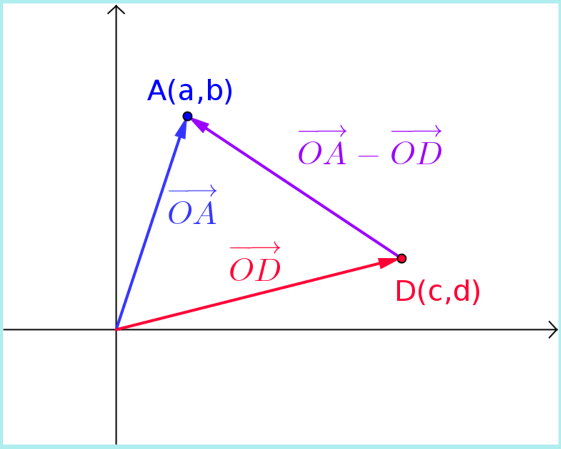
Difference of Vectors

The difference of two position vectors gives the vector between the original points (from the second to the first).



$$\begin{aligned}
\overrightarrow{OA}-\overrightarrow{OD} &= (a,b)-(c,d)\\
&= a\vec{i}+b\vec{j}-(c\vec{i}+d\vec{j})\\
&= a\vec{i}+b\vec{j}-c\vec{i}-d\vec{j}\\
&= (a-c)\vec{i}+(b-d)\vec{j}\\
&= (a-c,b-d)\\
&= \overrightarrow{DA}
\end{aligned}$$

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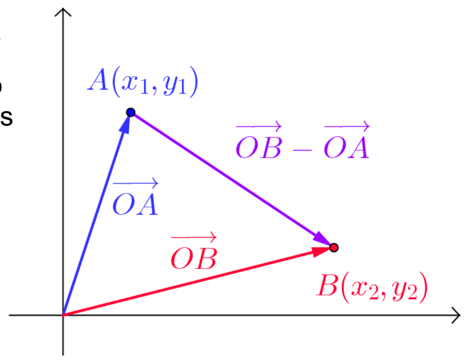

$$\begin{aligned}
\overrightarrow{AB} &= \overrightarrow{OB}-\overrightarrow{OA}\\
&= (x_2,y_2)-(x_1,y_1)\\
&= (x_2\vec{i}+y_2\vec{j})-(x_1\vec{i}+y_1\vec{j})\\
&= x_2\vec{i}+y_2\vec{j}-x_1\vec{i}-y_1\vec{j}\\
&= x_2\vec{i}-x_1\vec{i}+y_2\vec{j}-y_1\vec{j}\\
&= (x_2-x_1)\vec{i}+(y_2-y_1)\vec{j}\\
&= (x_2-x_1,y_2-y_1)
\end{aligned}$$

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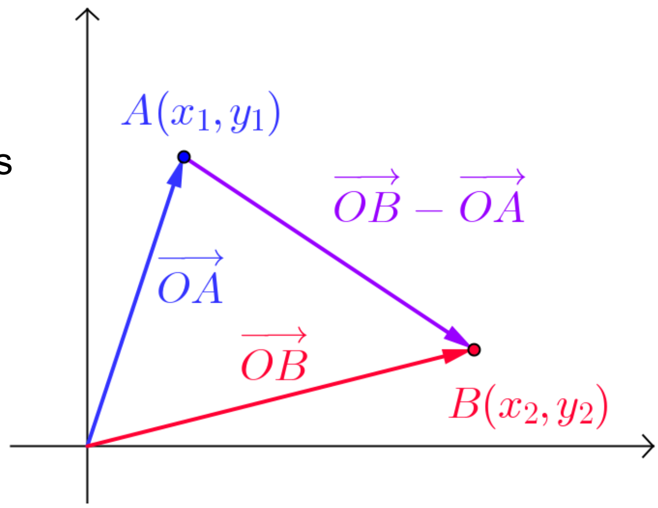

$$\left|\overrightarrow{AB}\right| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

which is the same as the distance formula between points A and B.

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Ex.1 Draw the following vectors in two dimensions.

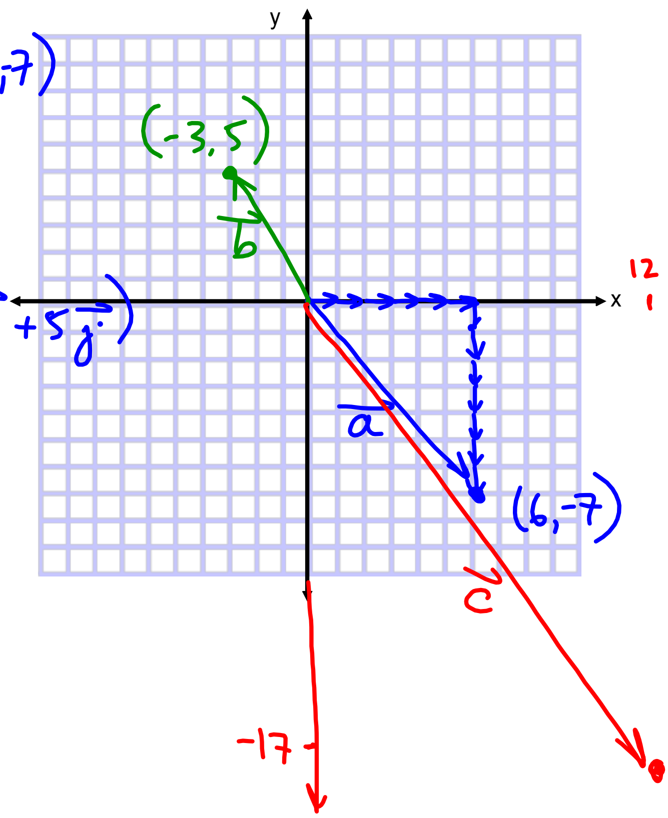
$\vec{a} = 6\vec{i} - 7\vec{j} = (6,-7)$

$\vec{b} = -3\vec{i} + 5\vec{j}$

$\vec{c} = \vec{a} - 2\vec{b}$

$= 6\vec{i} - 7\vec{j} - 2(-3\vec{i} + 5\vec{j})$

$= 12\vec{i} - 17\vec{j}$



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Ex.2 Given the points L(4,-2) and M(-2,3), find:
(a) $\overrightarrow{LM}$ and $|\overrightarrow{LM}|$
(b) a unit vector in the direction of $\overrightarrow{LM}$

$$\overrightarrow{LM} = \overrightarrow{OM} - \overrightarrow{OL}$$
$$= (-2,3) - (4,-2)$$
$$= (-2\vec{i} + 3\vec{j}) - (4\vec{i} - 2\vec{j})$$
$$= -6\vec{i} + 5\vec{j}$$
$$= (-6, 5)$$

$$|\overrightarrow{LM}| = \sqrt{(-6)^2 + (5)^2}$$
$$= \sqrt{61}$$

(b) $$\frac{\overrightarrow{LM}}{|\overrightarrow{LM}|} = \frac{(-6,5)}{\sqrt{61}}$$ ©

$$= \frac{-6\vec{i} + 5\vec{j}}{\sqrt{61}}$$ ok

$$= \frac{-6}{\sqrt{61}}\vec{i} + \frac{5}{\sqrt{61}}\vec{j}$$ ✓

$$= \left(\frac{-6}{\sqrt{61}}, \frac{5}{\sqrt{61}}\right)$$ ✓

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Ex.3 Given $\vec{a} = \vec{i} - 5\vec{j}$ and $\vec{b} = 4\vec{i} - 10\vec{j}$, find $|\vec{a} - 2\vec{b}|$.

NOTE: Solving problems using algebraic vectors (distance formula) can sometimes be simpler than solving them usinggeometric vectors (cosine law).

$$\vec{a} - 2\vec{b} = (\vec{i} - 5\vec{j}) - 2(4\vec{i} - 10\vec{j})$$
$$= -7\vec{i} + 15\vec{j}$$

$$|\vec{a} - 2\vec{b}| = \sqrt{(-7)^2 + (15)^2}$$
$$= \sqrt{49 + 225}$$
$$= \sqrt{274}$$

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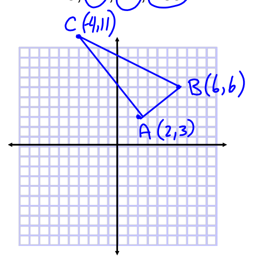
Assigned Work:

p.325 # 3, 4, 5b, 6ac, 7c, 8c, 10, 11, 12, 13a

11.



$|\vec{AB}| = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$

$= \sqrt{(6-2)^2 + (6-3)^2}$

$= \sqrt{16 + 9}$

$= 5$

$|\vec{AC}| = 10 \quad |\vec{CB}| = \sqrt{125}$

$\doteq 11.18$

(c) $|\vec{CB}|^2 = 125$

$|\vec{AC}|^2 + |\vec{AB}|^2$

$= 100 + 25$

$= 125$

Pythagorean relationship $c^2 = a^2 + b^2$

$\therefore$ right Δ

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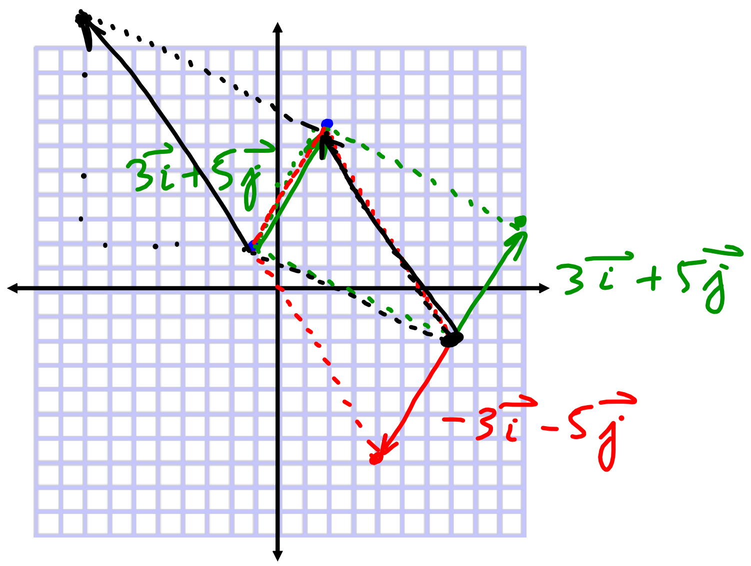
12



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13. (a) $3(x, 1) - 5(2, 3y) = (11, 33)$

$\vec{i}$ + $\vec{j}$ are at right angles

$\Rightarrow$ their components are independent of each other.

$$3x\vec{i} + 3\vec{j} - 10\vec{i} - 15y\vec{j} = 11\vec{i} + 33\vec{j}$$

$$(3x - 10 - 11)\vec{i} + (3 - 15y - 33)\vec{j} = 0\vec{i} + 0\vec{j}$$

$$3x - 10 - 11 = 0 \qquad 3 - 15y - 33 = 0$$

$$3x = 21 \qquad -30 = 15y$$

$$\boxed{x = 7} \qquad \boxed{y = -2}$$

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