

MCV4U Nelson p. 450 # 14.

$$L_1: x_1 = 4 + 2t, y_1 = 4 + 1t, z_1 = -3 - 1t, t \in \mathbb{R}$$

$$L_2: x_2 = -2 + 3s, y_2 = -7 + 2s, z_2 = 2 - 3s, s \in \mathbb{R}$$

P_1 on L_1 , P_2 on L_2 , $\overrightarrow{P_1P_2} \perp$ to L_1 and L_2

$$P_1(x_1, y_1, z_1)$$

$$P_2(x_2, y_2, z_2)$$

$$\vec{d}_1 = (2, 1, -1)$$

$$\vec{d}_2 = (3, 2, -3)$$

$$\begin{aligned}\vec{d}_3 &= \vec{d}_1 \times \vec{d}_2 \\ &= (-1, 3, 1)\end{aligned}$$

$$\begin{array}{cccc} \vec{i} & \vec{j} & \vec{k} & \vec{i} \\ 2 & 1 & -1 & 2 \\ 3 & 2 & -3 & 3 \end{array}$$

$$\begin{aligned}\overrightarrow{P_1P_2} &= r \vec{d}_3 \\ &= r(-1, 3, 1)\end{aligned}$$

$$\begin{aligned}\overrightarrow{P_1P_2} &= \overrightarrow{OP_2} - \overrightarrow{OP_1} \\ &= (x_2, y_2, z_2) - (x_1, y_1, z_1) \\ &= (x_2 - x_1, y_2 - y_1, z_2 - z_1)\end{aligned}$$

$$(x_2 - x_1, y_2 - y_1, z_2 - z_1) = r(-1, 3, 1)$$

$$x_2 - x_1 = -r$$

$$y_2 - y_1 = 3r$$

$$z_2 - z_1 = r$$

$$L_1: x_1 = 4 + 2t, y_1 = 4 + 1t, z_1 = -3 - t, t \in \mathbb{R}$$

$$L_2: x_2 = -2 + 3s, y_2 = -7 + 2s, z_2 = 2 - 3s, s \in \mathbb{R}$$

$$x_2 - x_1 = -r \quad y_2 - y_1 = 3r \quad z_2 - z_1 = r$$

$$(-2 + 3s) - (4 + 2t) = -r \quad -11 + 2s - t = 3r \quad 5 - 3s + t = r$$

$$-6 + 3s - 2t = -r \quad \textcircled{1} \quad -11 + 2s - t = 3r \quad \textcircled{2} \quad 5 - 3s + t = r \quad \textcircled{3}$$

$$-6 + 3s - 2(-1) = -r$$

$$-4 + 3s = -r \quad \textcircled{4}$$

$$-11 + 2s - (-1) = 3r$$

$$-10 + 2s = 3r \quad \textcircled{5}$$

$$\textcircled{4} \times 3: -12 + 9s = -3r$$

$$\textcircled{+} \quad -22 + 11s = 0$$

$$11s = 22$$

$$\boxed{s = 2}$$

$$-4 + 3(2) = -r$$

$$2 = -r$$

$$\boxed{r = -2}$$

$$x_1 = 4 + 2(-1)$$

$$= 2$$

$$y_1 = 4 + (-1)$$

$$= 3$$

$$z_1 = -3 - t$$

$$= -3 - (-1)$$

$$= -2$$

$$P_1(2, 3, -2)$$

$$P_2(4, -3, -4)$$