

Properties of Graphs of Functions

Sep. 4/2014

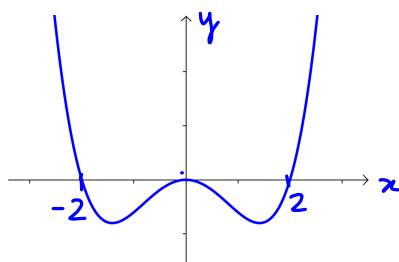
There are properties, or key features, of graphs of functions which can be used to classify and compare the functions involved.

- (1) domain and range
- (2) x-intercepts (zeroes) and y-intercept
- (3) intervals of increase or decrease
 - (i) on intervals of increase, y-values are increasing
 - (ii) on intervals of decrease, y-values are decreasing
 note: always read graph left-to-right, x-values increasing
- (4) turning points occur where functions change from increasing to decreasing, or vice versa
- (5) location of any discontinuities (e.g., asymptotes, holes), otherwise the function is continuous

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(6) Function Symmetry

- (a) Even symmetry can be seen graphically as a mirror image across the y-axis.



Any point (x,y) has a corresponding point $(-x,y)$

Algebraically:

$$f(-x) = f(x)$$

Ex.1 Show that $f(x) = x^2(x-2)(x+2)$, shown above, is an even function.

$$\begin{aligned}
 \text{LS} &= f(x) & \text{RS} &= f(-x) \\
 &= x^2(x-2)(x+2) & &= (-x)^2((-x)-2)((-x)+2) \\
 &= x^2(x^2-4) & &= x^2(-x-2)(-x+2) \\
 & & &= x^2(x^2-2x+2x-4) \\
 & & &= x^2(x^2-4) \\
 \text{LS} &= \text{RS} \\
 \therefore & \text{ even function}
 \end{aligned}$$

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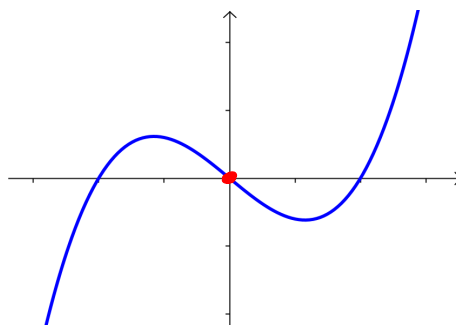
- (b) Odd symmetry is more difficult to see in the graph, as it represents a rotational, rather than reflective, symmetry about the origin.

Any point (x, y) has a corresponding point $(-x, -y)$.

Algebraically,

$$f(-x) = -f(x)$$

$$\text{or} \\ f(x) = -f(-x)$$



Ex.2 Show that $g(x) = x(x - 2)(x + 2)$ is an odd function.

$$\begin{aligned} \text{LS} &= g(-x) \\ &= (-x)(-x-2)(-x+2) \\ &= -x(x^2-4) \end{aligned} \qquad \begin{aligned} \text{RS} &= -g(x) \\ &= -[x(x-2)(x+2)] \\ &= -x(x^2-4) \end{aligned}$$

$$\text{LS} = \text{RS} \quad \therefore \text{odd function}$$

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- (7) End behaviour describes the tendency of the y-values as x-values approach very large positive and negative values (which we express abstractly as infinity).

$$\text{as } x \rightarrow \infty, y \rightarrow ?$$

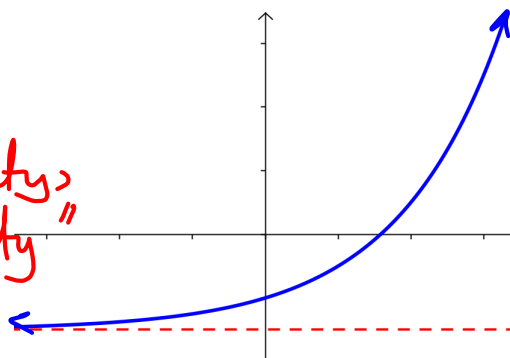
$$\text{as } x \rightarrow -\infty, y \rightarrow ?$$

Ex.3 Describe the end behaviour of $f(x) = 2^x - 3$

$$\text{as } x \rightarrow \infty, y \rightarrow \infty$$

"as x approaches infinity,
y approaches infinity"

$$\text{as } x \rightarrow -\infty, y \rightarrow -3$$



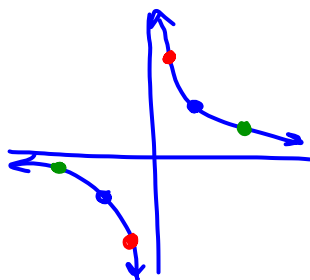
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Assigned Work:

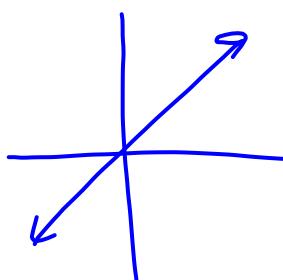
a
 a f b
 p.23 # 12, 3, 4, 5, 7, 8, 9, 10, 14
 use handout for 12, 14

$$4(a) \quad f(x) = \frac{1}{x}$$

$$g(x) = x$$

Common

odd

different
 domain, range
 $x \neq 0$

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$$5(f) \quad f(x) = |2x+3|$$

$$f(-x) = |2(-x)+3|$$

$$\begin{aligned}
 f(-x) &= |-2x+3| \\
 &= |-(2x-3)| \\
 &= |2x-3|
 \end{aligned}$$

$$\begin{aligned}
 -f(x) &= -(|2x+3|) \\
 &= -(2x+3)
 \end{aligned}$$

$$\begin{aligned}
 \underbrace{-f(-x)}_{\text{odd}} &= -|-2x+3| \\
 -f(-x) &= f(x) \quad f(x) = |2x+3| \\
 &= -|(-1)(2x-3)| \quad |(-1)x| \\
 &= -|2x-3| \quad \therefore \text{not odd}
 \end{aligned}$$

even: $f(x) = f(-x)$
 \therefore not even
 $\therefore f(x)$ is neither.

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8(a)

interval

$$x > 2 \rightarrow (2, \infty)$$

↑ does not include $x=2$ ↑ never equal to infinity

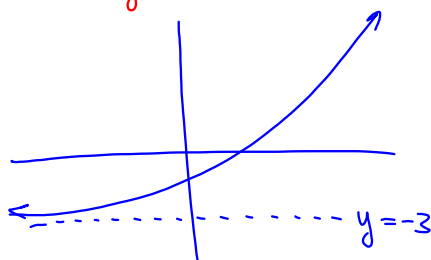
$$x \in \mathbb{R} \rightarrow (-\infty, \infty)$$

$$x \leq 2 \rightarrow (-\infty, 2]$$

↑ includes $x=2$

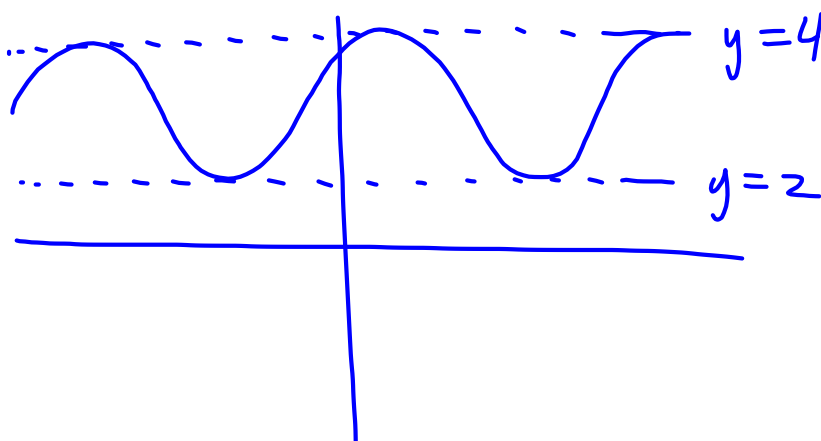
$$D = \{x \in \mathbb{R}\}, \text{ INC } (-\infty, \infty)$$

$$R = \{ \underset{y}{f(x)} \in \mathbb{R} \mid f(x) > -3 \}$$

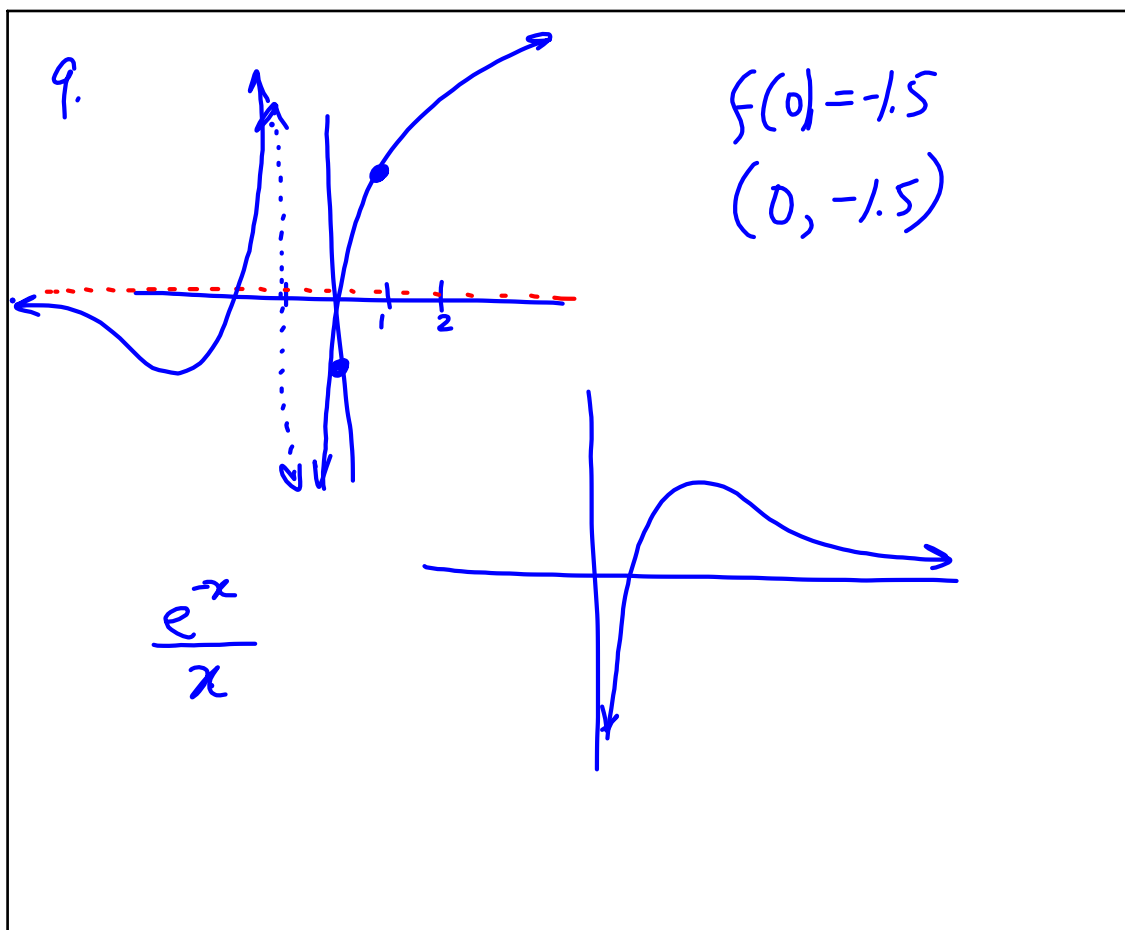


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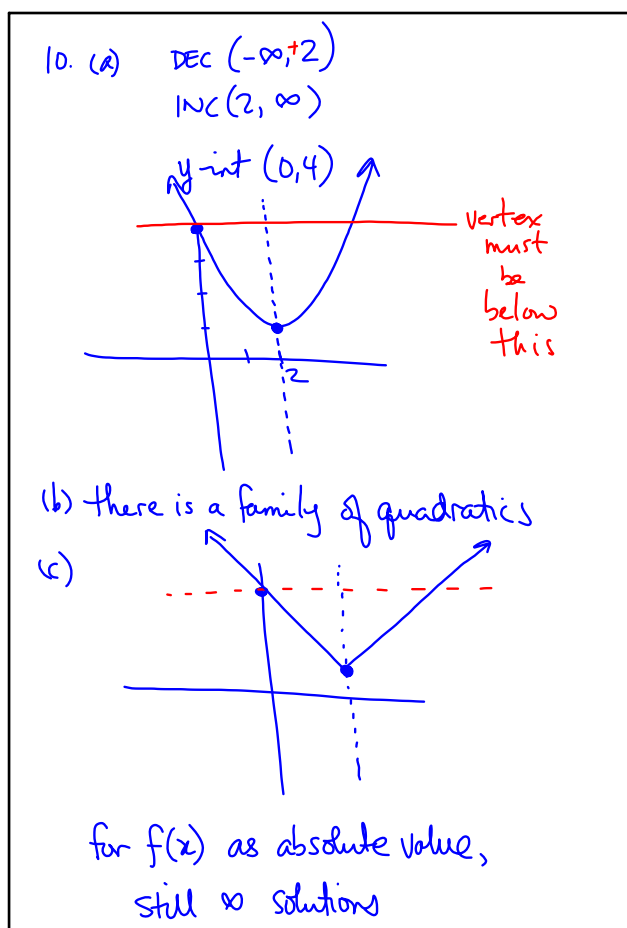
$$8(b) \quad R = \{ g(x) \in \mathbb{R}, 2 \leq g(x) \leq 4 \}$$



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Attachments

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