Properties of Graphs of Functions

Sept. 4/2014

There are properties, or key features, of graphs of functions which can be used to classify and compare the functions involved.

(1) domain and range
(2) x-intercepts (zeroes) and y-intercept

(3) intervals of increase or decrease
  (i) on intervals of increase, y-values are increasing
  (ii) on intervals of decrease, y-values are decreasing
  note: always read graph left-to-right, x-values increasing

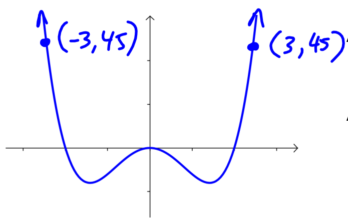
(4) turning points occur where functions change from increasing to decreasing, or vice versa

(5) location of any discontinuities (e.g., asymptotes, holes), otherwise the function is continuous

Sep 2-8:51 PM

(6) Function Symmetry

(a) Even symmetry can be seen graphically as a mirror image across the y-axis.



Any point (x,y) has a corresponding point (-x,y)

Algebraically:

$$f(-x) = f(x)$$

Ex.1 Show that $f(x) = x^2(x-2)(x+2)$, shown above, is an <u>even function</u>.

$LS = f(-x)$
$= (-x)^2((-x)-2)((-x)+2)$
$= x^2(x^2-2x+2x-4)$
$= x^2(x^2-4)$

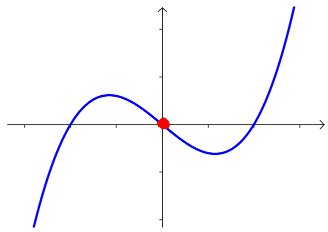
$RS = f(x)$
$= x^2(x-2)(x+2)$
$= x^2(x^2-4)$

$LS = RS$
$\therefore f(x)$ is even

Sep 3-8:30 PM

(b) Odd symmetry is more difficult to see in the graph, as it represents a rotational, rather than reflective, symmetry about the origin.

Any point (x,y) has a corresponding point (-x,-y).



Algebraically,

$$f(-x) = -f(x)$$

OR

$$-f(-x) = f(x)$$

Ex.2 Show that $g(x) = x(x-2)(x+2)$ is an odd function.

$LS = -g(-x)$

$= -1[(-x)((-x)-2)((-x)+2)]$

$= -[(-x)(x^2-4)]$

$= +x(x^2-4)$

$RS = x(x-2)(x+2)$

$= x(x^2-4)$

$LS = RS \therefore g(x)$ is odd

Sep 3-8:44 PM

(7) End behaviour describes the tendency of the y-values as x-values approach very large positive and negative values (which we express abstractly as infinity).

$$\text{as } x \to \infty,\ y \to ?$$

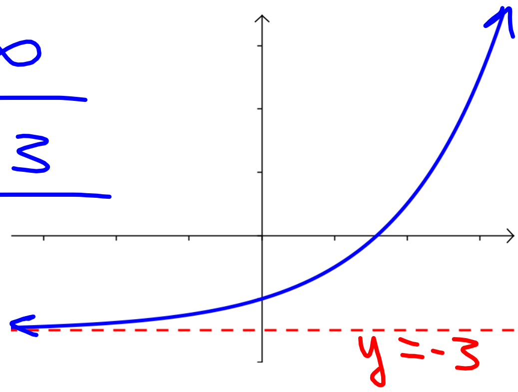
$$\text{as } x \to -\infty,\ y \to ?$$

Ex.3 Describe the end behaviour of $f(x) = 2^x - 3$

"approaches"

as $x \to \infty$, $y \to$ $\underline{\infty}$

as $x \to -\infty$, $y \to$ $\underline{-3}$



Sep 3-9:08 PM

Assigned Work:

p.23 # 12, 3, 4, 5, 7, 8, 9, 10, 14
use handout for 12, 14

5bdc

10

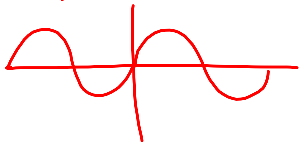
8

$f(90) = 91$

5(b) $f(x) = \sin x + x$

$f(-90) = -91$

$\sin(x) \neq \sin(-x)$



even: $f(x) = f(-x)$

odd: $f(-x) = -f(x)$

or

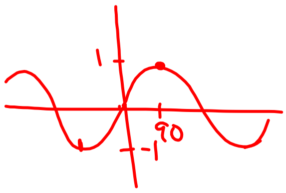
$f(x) = -f(-x)$

$f(-x) = \sin(-x) - x$

$-f(x) = -[\sin x + x]$

$= -\sin x - x$

$= \sin(-x) - x$



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5(c) $f(x) = \frac{1}{x} - x$

$f(-x) = \frac{1}{-x} - (-x)$

$= \frac{-1}{x} + x$

$f(x) \neq f(-x)$ $\therefore$ not even

$-f(x) = -\left(\frac{1}{x} - x\right)$

$= \frac{-1}{x} + x$

$f(-x) = -f(x)$ $\therefore$ odd

$-f(-x) = -\left[\frac{1}{-x} - (-x)\right]$

$= -\left[\frac{-1}{x} + x\right]$

$= \frac{1}{x} - x$

$f(x) = -f(-x)$ $\therefore$ odd

Sep 5-12:42 PM

5(d) $f(x) = 2x^3 + x$

$(-x)(-x)(-x)$

even?: $f(-x) = 2(-x)^3 + (-x)$

NO

$= -2x^3 - x$

odd: $-f(x) = -(2x^3 + x)$

$= -2x^3 - x$

$f(-x) = -f(x)$

$\therefore$ odd function

Sep 5-12:45 PM

interval notation

$2 < x \leq 3 \longrightarrow (2, 3]$

does not include 2

between 2 + 3, includes 3

does include 3

$x > -5$ $(-5, \infty)$

does not include 5

∞ is not a number, so always round bracket

$(-\infty, \infty) \longrightarrow x \in \mathbb{R}$

$x \leq 3 \longrightarrow (-\infty, 3]$

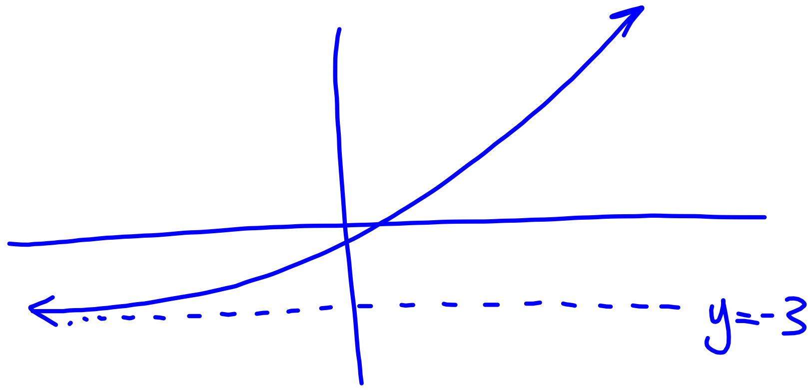
Sep 5-12:47 PM

8. (a) $D = \{x \in \mathbb{R}\}$

INC $(-\infty, \infty)$

$R = \{ f(x) \in \mathbb{R} \mid f(x) > -3 \}$

$y \in \mathbb{R}$ $\quad$ $y > -3$



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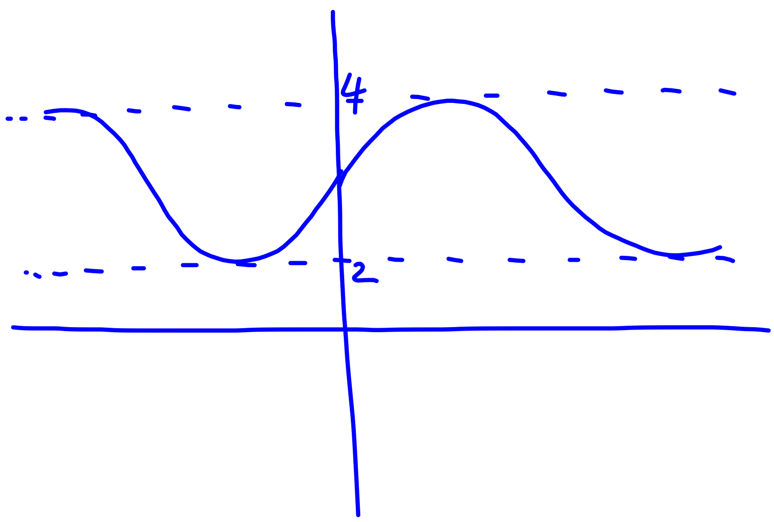
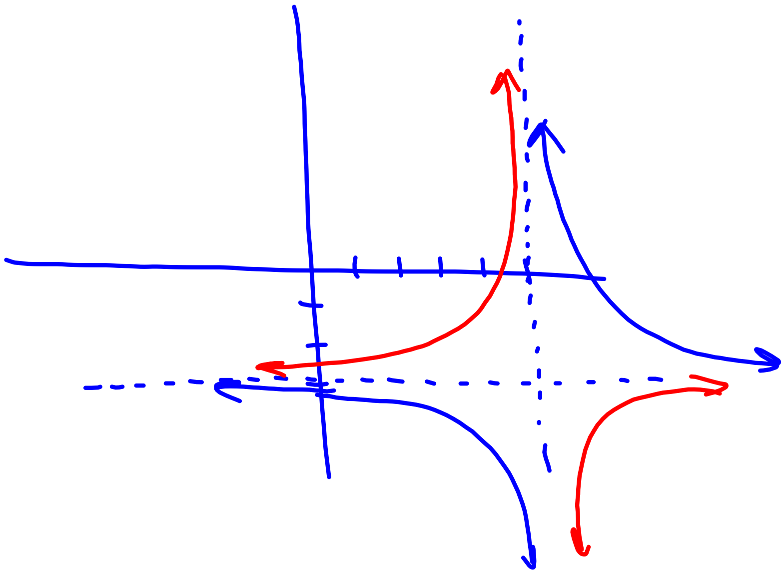
8(b) $R = \{ g(x) \in \mathbb{R} \mid 2 \leq x \leq 4 \}$



Sep 5-12:59 PM

8(c) $D = \{x \in \mathbb{R} \mid x \neq 5\}$

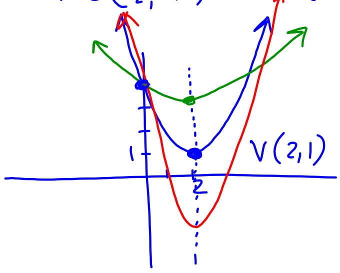
$R = \{h(x) \in \mathbb{R} \mid h(x) \neq -3\}$



Sep 5-1:01 PM

10 DEC $(-\infty, 2)$

INC $(2, \infty)$ y-int $(0,4)$



$f(x) = a(x-p)^2 + k$

$= a(x-2)^2 + 1$

sub $f(0) = 4$ y-int $(0,4)$

$4 = a(-2)^2 + 1$

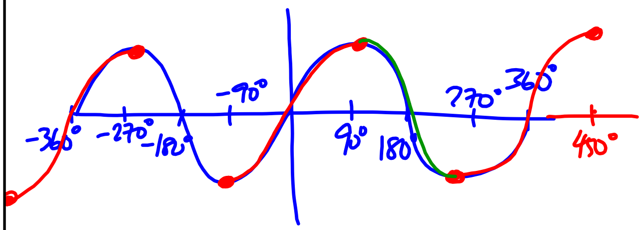
$4 = 4a + 1$

$3 = 4a$

$a = \frac{3}{4}$ $f(x) = \frac{3}{4}(x-2)^2 + 1$

Sep 5-1:03 PM

Handout - sinusoidal



int of inc: $(-90°, 90°)$

$(270°, 450°)$

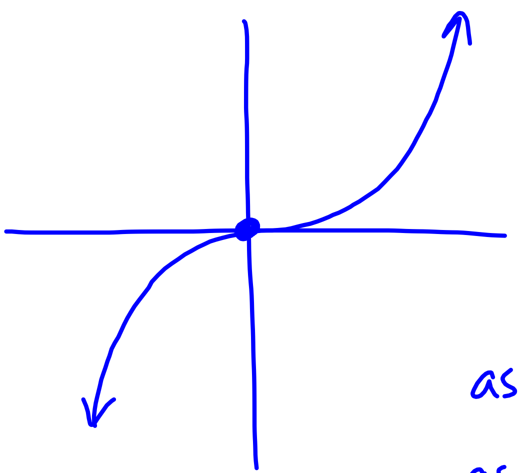
$(630°, 810°)$

$(-90° + 360°n, 90° + 360°n), n \in \mathbb{Z}$

int of dec: $(90° + 360°n, 270° + 360°n), n \in \mathbb{Z}$ $n \in \mathbb{I}$

Sep 5-1:10 PM

Handout - cubic



as $x \to \infty$, $y \to \infty$

as $x \to -\infty$, $y \to -\infty$

Sep 5-1:19 PM

Attachments

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