

Inverse Functions (review)

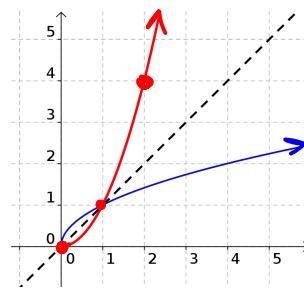
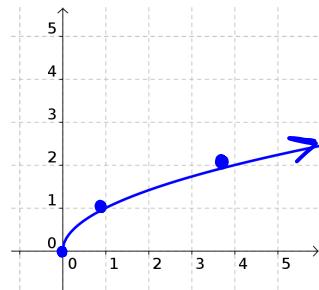
Sept 8/2014

The inverse of a relation can be found by interchanging the domain and range of the relation (i.e., swap x and y).

	<u>original relation</u>	<u>inverse relation</u>
points:	$\{(a, b), (c, d)\}$	$\{(b, a), (d, c)\}$

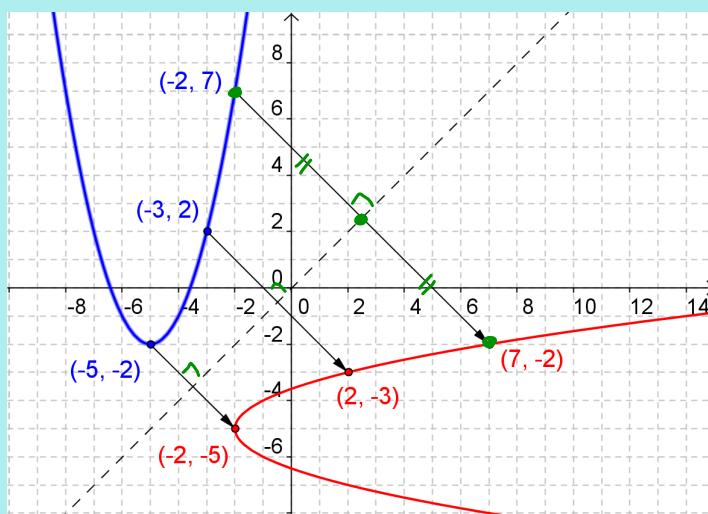
equation:	express in terms of independent and dependent variables	swap x and y, rearrange to $y = \dots$
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graph:

reflect in line $y = x$

Feb 22-9:25 PM

Find the inverse of $y = (x + 5)^2 - 2$ graphically



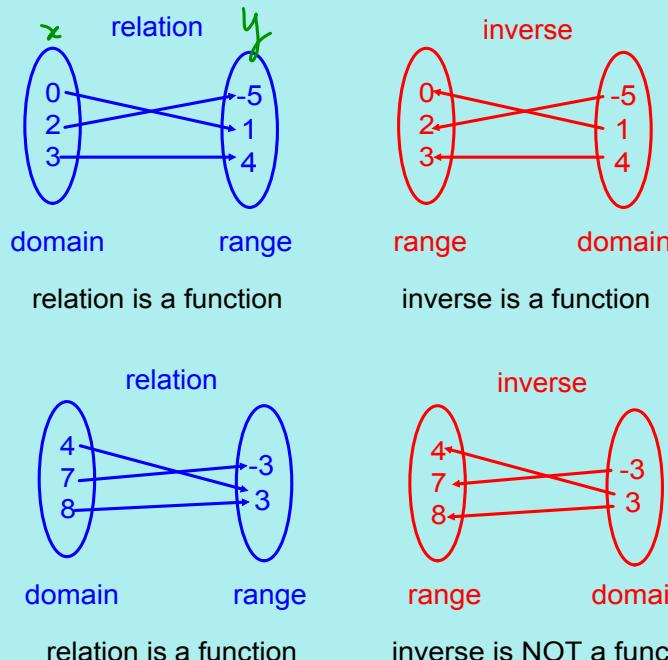
Notice that the original points and the reflected (swapped) points are equidistant (equal distance) to the line $y = x$.

The inverse (red) fails the vertical line test, and is not a function.

Feb 23-9:20 PM

A mapping diagram can be used to determine if a relation is a function.

If there is only one arrow from each item in the domain, then it is a function.



Feb 22-8:37 PM

Recall: A function is a special type of relation where each element in the domain corresponds to a single value in the range.

For an inverse function, each value in the range corresponds to a single value in the domain.

If the inverse of the function, $f(x)$, is also a function, it is given the special designation of inverse function, $f^{-1}(x)$

Note: In the inverse notation, the "-1" is not an exponent!

For example:

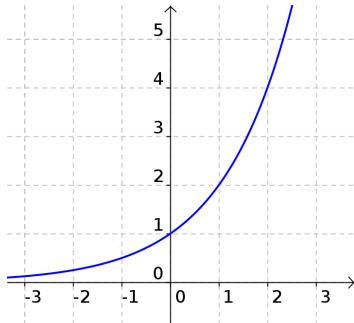
$$x^{-1} = \frac{1}{x} \quad f^{-1}(x) \neq \frac{1}{f(x)}$$

Feb 22-10:25 PM

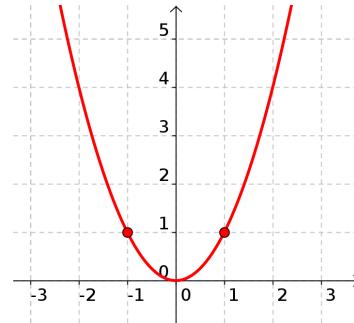
A function and its inverse function undo each other.

Given $f(a) = b$, then $f^{-1}(b) = a$
(assuming the inverse is a function)

The inverse of a function will also be a function if each x-value produces a unique y-value.



each x produces a unique y-value, inverse is a function



each x produces a single y-value, but they are not unique: not a function
inVerse

Feb 22-9:04 PM

The inverse of a function may not be a function. It is possible to restrict the domain to force the inverse to be a function.

of $f(x)$

Ex.1 Find the inverse of $f(x) = 3x^2 - 6$

$$\begin{aligned}
 & \text{for inverse,} \\
 & \textcircled{O} \text{ Swap } x, y: x = 3y^2 - 6 \\
 & y = 3x^2 - 6 \\
 & x + 6 = 3y^2 \\
 & y^2 = \frac{x+6}{3} \\
 & y = \pm \sqrt{\frac{x+6}{3}} \\
 & y = \pm \sqrt{\frac{1}{3}(x+6)}
 \end{aligned}$$

restriction of $f(x)$: $D_f = \{x \in \mathbb{R} \mid x \geq 0\}$

OR
 $D_f = \{x \in \mathbb{R} \mid x \leq 0\}$

Feb 22-10:21 PM

Assigned Work:

p.44 # 4, 5, 6bd, 7, 9, 11, 12c, 13, 17

a
b

d

$$4(a) \quad f(x) = 2x^3 + 1 \quad f(4) = 2(4)^3 + 1$$

$$(4, f(4)) = (4, 129)$$

$$= (4, 129)$$

(b) inverse, $f^{-1}(129, 4)$

Feb 10-10:23 PM

$$7. \quad F = \frac{9}{5}C + 32$$

$$F(C) = \frac{9}{5}C + 32$$

$$\begin{array}{ccc} x & \rightarrow & \boxed{f(x)} \rightarrow y \\ C & \rightarrow & \boxed{F(C)} \rightarrow F \end{array}$$

for inverse, don't swap F, C
→ rearrange equation

$$F = \frac{9}{5}C + 32$$

$$F - 32 = \frac{9}{5}C$$

$$\frac{5}{9}(F - 32) = C$$

$$C(F) = \frac{5}{9}(F - 32)$$

$$= \frac{5}{9}F - \frac{160}{9}$$

$$f(x) = \frac{9}{5}x + 32$$

(x is temp in $^{\circ}\text{C}$,
 $f(x)$ is temp in $^{\circ}\text{F}$)

$$y = \frac{9}{5}x + 32$$

Swap x, y

$$x = \frac{9}{5}y + 32$$

$$y = \frac{5}{9}(x - 32)$$

$$f^{-1}(x) = \frac{5}{9}(x - 32)$$

(x is temp in $^{\circ}\text{F}$
 $f^{-1}(x)$ is temp in $^{\circ}\text{C}$)

Sep 9-10:30 AM

9. $f(x) = kx^3 - 1$ $f^{-1}(15) = 2$

$f(2) = k(2)^3 - 1$ $f(2) = 15$

$15 = 8k - 1$

$16 = 8k$

$k = 2$

$\sin(30^\circ) = \frac{1}{2}$

$\sin^{-1}(\frac{1}{2}) = 30^\circ$

$y = kx^3 - 1$

Swap x, y

$x = ky^3 - 1$

$x + 1 = ky^3$

$\frac{x+1}{k} = y^3$

$y = \sqrt[3]{\frac{x+1}{k}}$

$f^{-1}(x) = \sqrt[3]{\frac{x+1}{k}}$

$f^{-1}(15) = \sqrt[3]{\frac{15+1}{k}}$

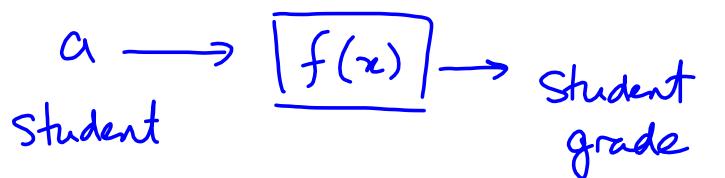
$(2) = \left(\sqrt[3]{\frac{16}{k}} \right)^3$

$8 = \frac{16}{k}$

$k = 2$

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11.

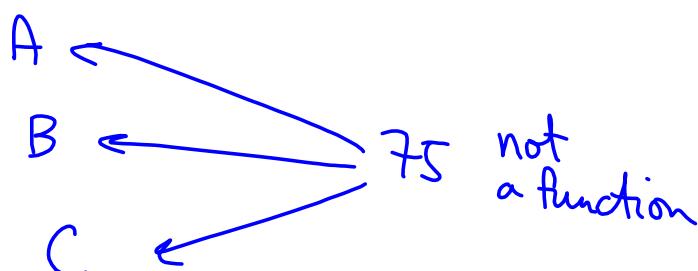


A \longrightarrow 75

B \longrightarrow 75

C \longrightarrow 75

is a function



not a function

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13(d) $g(x) = 4(x-3)^2 + 1$

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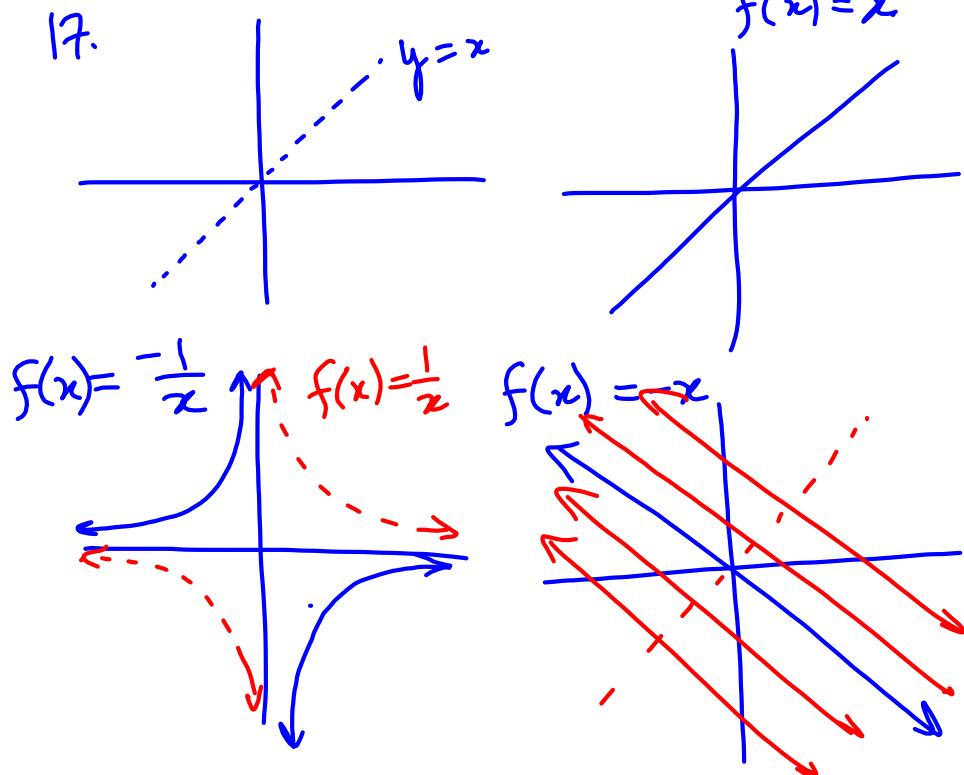
$$y = \pm \sqrt{\frac{x-1}{4}} + 3$$

$$4(x-3)^2 + 1 = \sqrt{\frac{x-1}{4}} + 3$$

$$4(x^2 - 6x + 9) - 2 = \sqrt{\frac{x-1}{4}}$$

Sep 9 10:53 AM

17.



Sep 9 10:56 AM