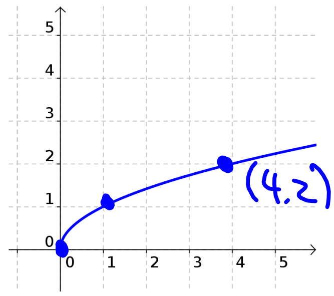
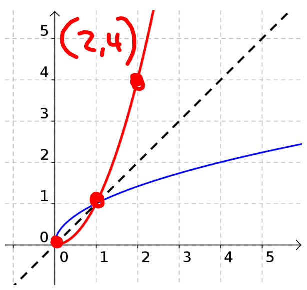
Inverse Functions (review)

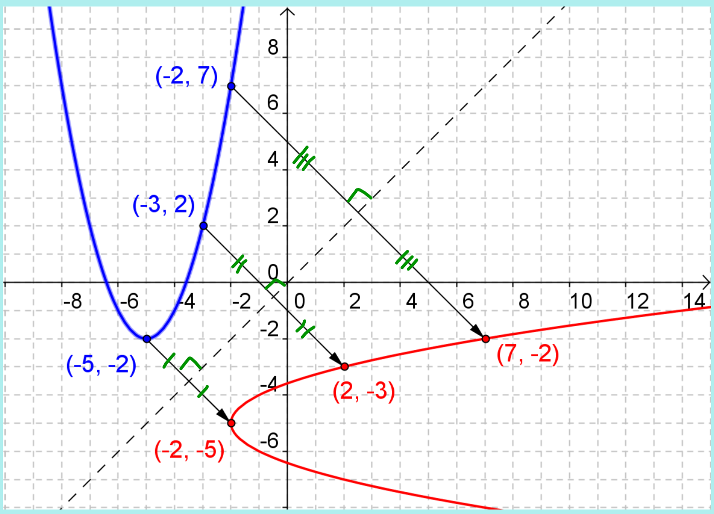
Sept 8/2014

The inverse of a relation can be found by interchanging the domain and range of the relation (i.e., swap x and y).

	original relation	inverse relation
points:	$\{(a,b),(c,d)\}$	$\{(b,a),(d,c)\}$
equation:	express in terms of independent and dependent variables	swap x and y, rearrange to $y = \ldots$
graph:		 reflect in line y = x

Feb 22-9:25 PM

Find the inverse of $y=(x+5)^2-2$ graphically



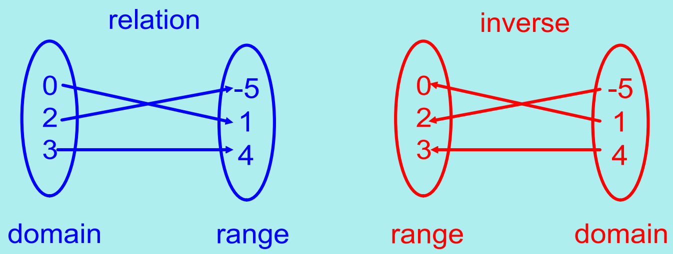
Notice that the original points and the reflected (swapped) points are equidistant (equal distance) to the line y = x.

The inverse (red) fails the vertical line test, and is <u>not</u> a function.

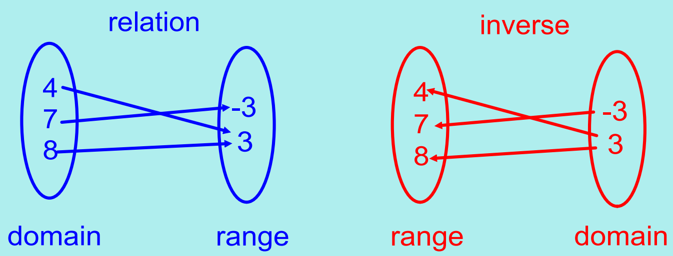
Feb 23-9:20 PM

A mapping diagram can be used to determine if a relation is a function.

If there is only one arrow from each item in the domain, then it is a function.



relation is a function   inverse is a function



relation is a function   inverse is NOT a function

Feb 22-8:37 PM

Recall: A function is a special type of relation where each element in the domain corresponds to a single value in the range.

For an inverse function, each value in the range corresponds to a single value in the domain.

If the inverse of the function, $f(x)$, is also a function, it is given the special designation of inverse function, $f^{-1}(x)$

Note: In the inverse notation, the "-1" is not an exponent!

For example: $x^{-1} = \dfrac{1}{x}$ $f^{-1}(x) \neq \dfrac{1}{f(x)}$

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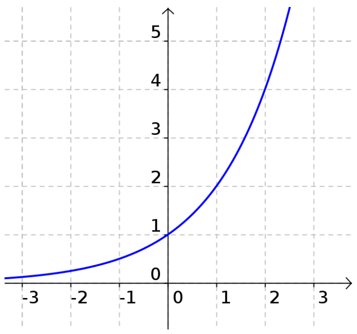
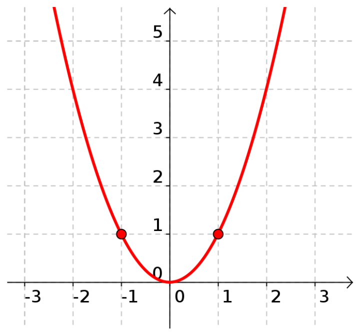
A function and its inverse function <u>undo</u> each other.

Given $f(a) = b$, then $f^{-1}(b) = a$
(assuming the inverse is a function)

The inverse of a function will also be a function if each x-value produces a <u>unique</u> y-value.



each x produces a unique y-value, inverse is a function



each x produces a single y-value, but they are not unique; not a function
inverse is

Feb 22-9:04 PM

The inverse of a function may not be a function. It is possible to <u>restrict the domain</u> to force the inverse to be a function.

Ex.1 Find the inverse of $f(x) = 3x^2 - 6$

$y = 3x^2 - 6$

swap x, y

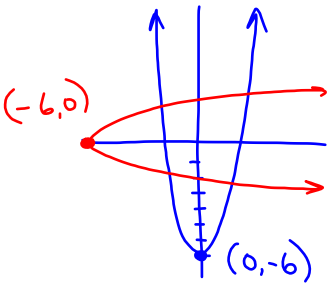
$x = 3y^2 - 6$

$x + 6 = 3y^2$

$y^2 = \frac{x+6}{3}$

$y = \pm\sqrt{\frac{x+6}{3}}$

$y = \pm\sqrt{\frac{x}{3} + 2}$



keep <u>top</u> or bottom half

$D_f = \{x \in \mathbb{R} \mid x \geq 0\}$

keep right side of $f(x)$

$f^{-1}(x) = \sqrt{\frac{x+6}{3}}$

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Assigned Work:

p.44 # 4, 5, 6bd, 7, 9, 11, 12c, 13, 17

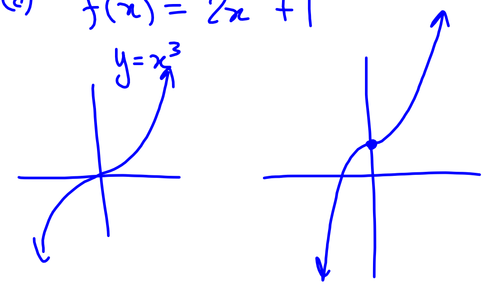
b, c, d a

$$f(x) = 2x^3 + 1$$

4(b) $(4, f(4)) \rightarrow (4, 129)$

for inverse, swap x, y $P(129, 4)$

(c) $f(x) = 2x^3 + 1$

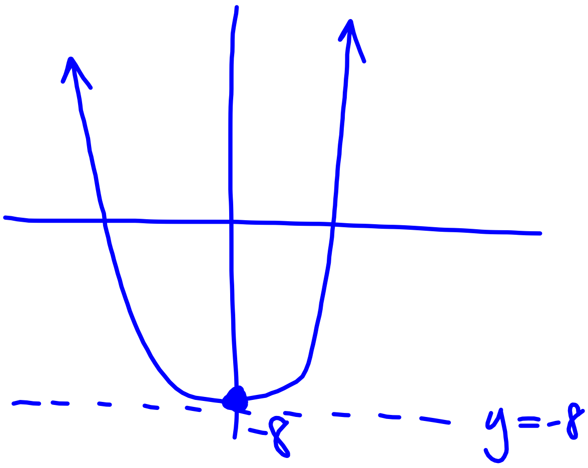


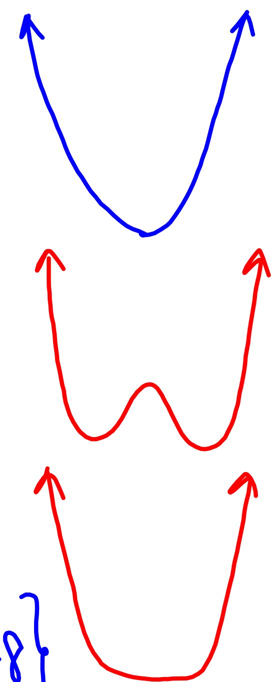
$f(x)$: $D = \{x \in \mathbb{R}\}$ $R = \{y \in \mathbb{R}\}$

(d) inverse: $D = \{x \in \mathbb{R}\}$ $R = \{y \in \mathbb{R}\}$

Feb 10-10:23 PM

5. $g(x) = x^4 - 8$





$D = \{x \in \mathbb{R}\}$ $R = \{y \in \mathbb{R} \mid y \geq -8\}$

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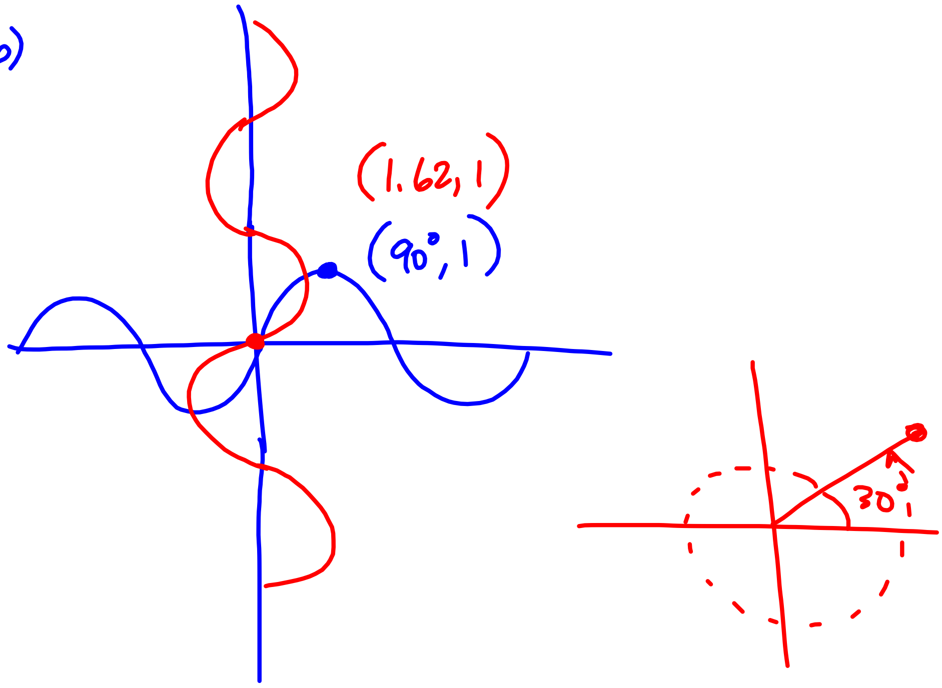
6(b)



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7(a) $F = \frac{9}{5}C + 32$

for inverse, DO NOT swap F & C, because they have clear definitions

→ rearrange

$\frac{5}{9}(F-32) = (\cancel{\frac{9}{5}}C)\cancel{\frac{5}{9}}$

$C = \frac{5}{9}(F-32)$

b) $F(20) = \frac{9}{5}(20) + 32$

$= 68$

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9. $f(x) = kx^3 - 1$

$f(2) = k(2)^3 - 1$

$15 = 8k - 1$

$16 = 8k$

$k = 2$

$f^{-1}(15) = 2$

$f(2) = 15$

$\sin 30° = \frac{1}{2}$

$\sin^{-1}\left(\frac{1}{2}\right) = 30°$

$y = kx^3 - 1$

Swap x, y

$x = ky^3 - 1$

$y = \sqrt[3]{\frac{x+1}{k}}$

$f^{-1}(x) = \sqrt[3]{\frac{x+1}{k}}$

$f^{-1}(15) = \sqrt[3]{\frac{15+1}{k}}$

$(2)^3 = \left(\sqrt[3]{\frac{16}{k}}\right)^3$

$8 = \frac{16}{k}$

$k = 2$

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11.

student $a \rightarrow \boxed{f(a)} \rightarrow$ student average

$A \longrightarrow 80$

$B \longrightarrow 70$

$C \longrightarrow 80$

$f(x)$ is a function

$A \leftarrow 80 \rightarrow C$

Inverse is not

Sep 9-1:00 PM

$g(x) = 4(x-3)^2 + 1$

(a) $y = 4(x-3)^2 + 1$

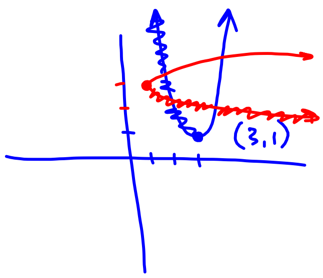
swap

$x = 4(y-3)^2 + 1$

(b) $x - 1 = 4(y-3)^2$

$\pm\sqrt{\frac{x-1}{4}} = y - 3$ or $y = \pm\sqrt{\frac{x-1}{4}} + 3$

$y = \pm\frac{\sqrt{x-1}}{2} + 3$



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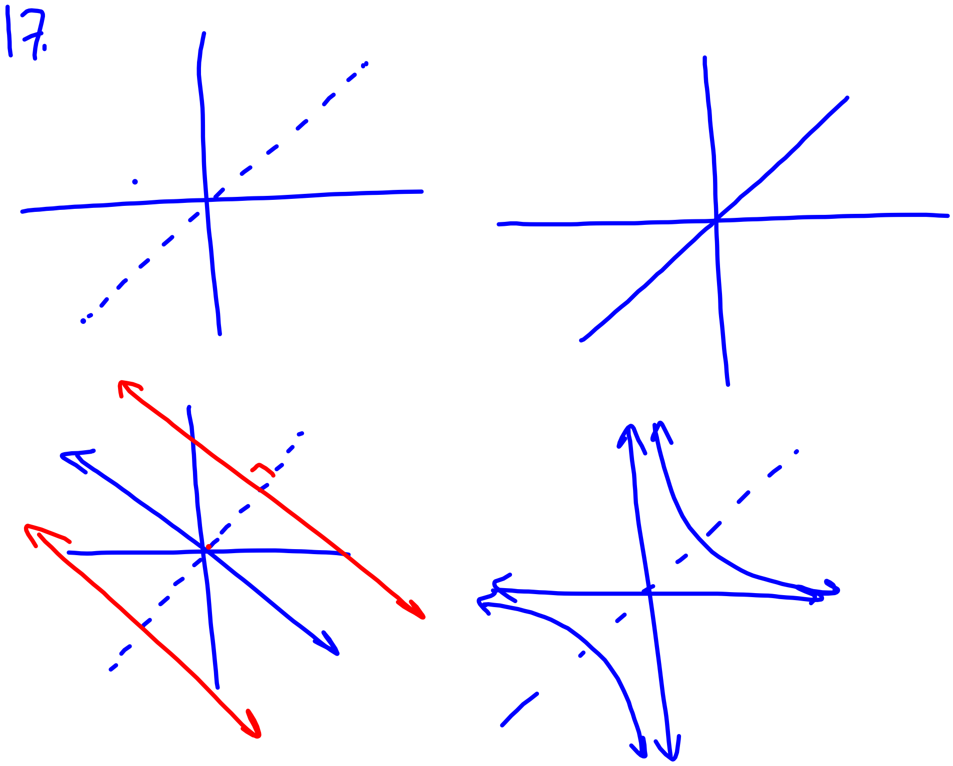
17.



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