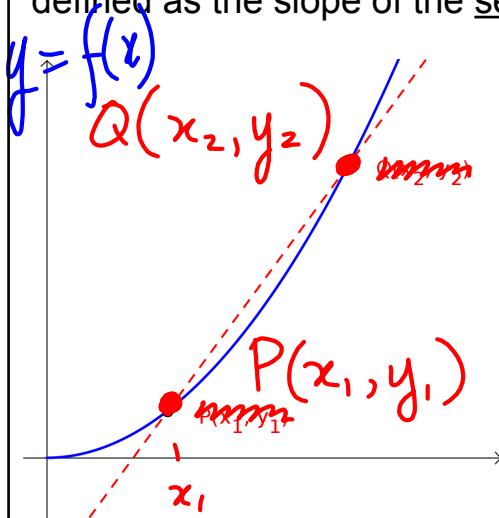


Rates of Change

Sept 10/2014

Given the graph of a function, the average rate of change is defined as the slope of the secant line between two points.



$$\begin{aligned}\text{avg RoC} &= m_{PQ} \\ &= \frac{\Delta y}{\Delta x} \\ &= \frac{y_2 - y_1}{x_2 - x_1}\end{aligned}$$

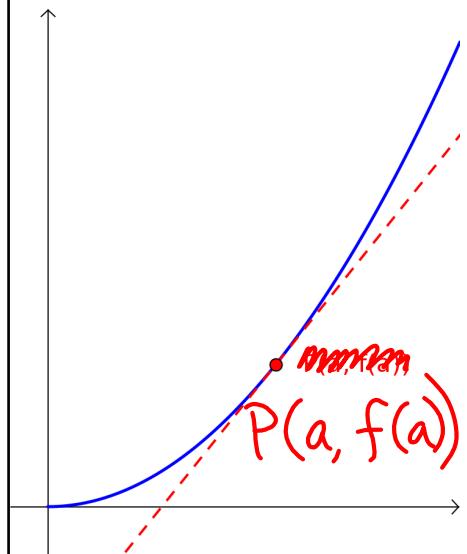
Using function notation, the points can also be written:

$$\begin{aligned}P(x_1, f(x_1)) \\ Q(x_2, f(x_2))\end{aligned}$$

$$\therefore \text{avg RoC} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

Sep 9-8:15 PM

The instantaneous rate of change is the slope of the tangent line at a particular point of interest, defined by a specified value of the independent variable (e.g., at  $x = a$ ).



For now, we can only estimate this value by determining the average rate of change over a very small interval near  $x = a$ .

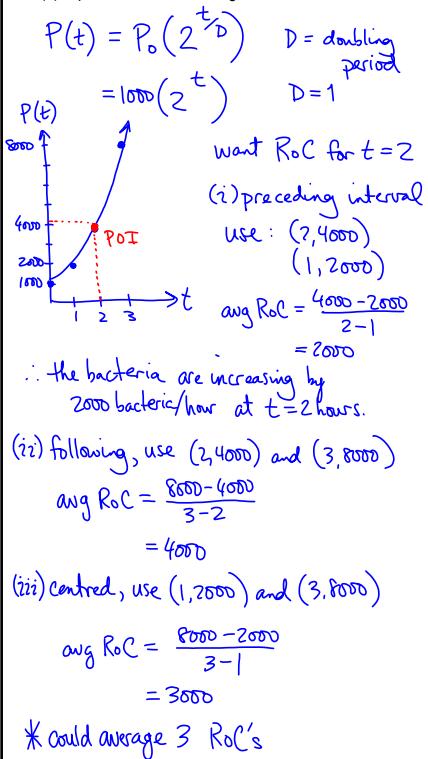
- (a) a preceding interval uses a point before the point of interest.
- (b) a following interval uses a point after the point of interest.
- (c) a centred interval uses points on either side of the point of interest.

Sep 9-8:57 PM

Ex.1 A bacterial colony starts with 1000 bacteria and doubles each hour.

- (a) Estimate the growth rate (bacteria/hour) after 2 hours using 1 hour intervals  
 (i) preceding  
 (ii) following  
 (iii) centred

- (b) improve the estimate using 0.1 hour intervals



Sep 9-9:09 PM

(b) set interval to 0.1 hours

(i) preceding  $(2, 4000)$   $(1.9, P(1.9))$   $\left\{ \begin{array}{l} P(t) = 1000(2^t) \\ P(1.9) = 1000(2^{1.9}) \end{array} \right. \therefore 3732$

$= (1.9, 3732)$

$\text{avg RoC} = \frac{4000 - 3732}{2 - 1.9}$   
 $= \frac{268}{0.1}$   
 $= 2680$

(ii) following  $(2, 4000)$   $P(t) = 1000(2^t)$   
 $(2.1, 4287)$   $P(2.1) = 1000(2^{2.1})$   
 $\therefore$

$\text{avg RoC} = \frac{4287 - 4000}{2.1 - 2}$   
 $\therefore 2870$

(iii) centred on  $x=2$

0.1 wide  $\left\{ \begin{array}{l} (1.95, 3863) \\ (2.05, 4141) \end{array} \right. \quad \text{avg RoC} = \frac{4141 - 3863}{2.05 - 1.95}$   
 $\therefore 2780$

estimate  $\therefore \frac{2680 + 2870 + 2780}{3}$   
 $\therefore 2777$

Sep 10-3:02 PM

In general, we algebraically represent the estimated instantaneous rate of change as a difference quotient.

For  $x = a$ , the point of interest is  $P(a, f(a))$

The following point occurs at  $x = a + h$ , where  $h$  is an arbitrarily small value, giving a second point

no fixed rule, choose  
Value best estimate

$Q(a + h, f(a + h))$

$$\begin{aligned}\text{avg RoC} &= m_{PQ} \\ &= \frac{f(a+h) - f(a)}{(a+h) - a} \\ &= \frac{f(a+h) - f(a)}{h}\end{aligned}$$

Sep 9-9:26 PM

To estimate instantaneous rate of change:

- (a) Use a series of preceding and following intervals, keeping the point of interest constant. As the intervals get smaller and smaller, look for the trend in values.
- (b) Use a series of centred intervals and look for the trend.
- (c) Use the difference quotient for very small values of  $h$  (both positive and negative work).

The best estimates come from the smallest intervals.

Sep 9-9:35 PM

Assigned Work:

p.76 # 8, 9  
 p.85 # 4, 7, 9, 10, 15  
 (5)

p.76 8(a) (i) +750 people/year  
 (ii) +2000 people/year

(b) not constant

(c) assume doubling every 10 years  
 use  $t = 100$

Sep 9-9:41 PM

p.76 #9,

$$h(t) = 18t - 0.8t^2 \quad P_1(10, h(10))$$

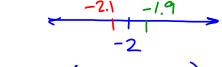
$$10 \leq t \leq 15 \quad P_2(15, h(15))$$

$$\text{avg RoC} = \frac{h(15) - h(10)}{15 - 10}$$

$$= \underline{\hspace{2cm}}$$

Sep 11-1:02 PM

P.86 4(a)  $f(x) = 6x^2 - 4$   
 $x = -2$

start  $h = 0.1$  

use  $(-2, h(-2))$  and  $(-2.1, h(-2.1))$   
 $(-2, 20)$        $(-2.1, 22.46)$

$\text{avg RoC} = \frac{22.46 - 20}{-2.1 - (-2)}$   
 $= \frac{2.46}{-0.1}$   
 $= -24.6$

try  $h = 0.01$   $(-2, 20)$   
 $(-2.01, 20.02406)$

$\text{avg RoC} = \frac{20.02406 - 20}{-0.01}$   
 $= -24.06$

try  $h = 0.001$   $(-2.001, 20.0024006)$

$\text{avg RoC} = -24.006$

$\Rightarrow$  trend seems to be toward -24

$\therefore$  estimate of inst RoC is -24

Sep 11-1:05 PM

5.  $h(x) = -5x^2 + 3x + 65$

i RoC for  $x = 3$

try  $h = 0.01$

and  $h = 0.001$

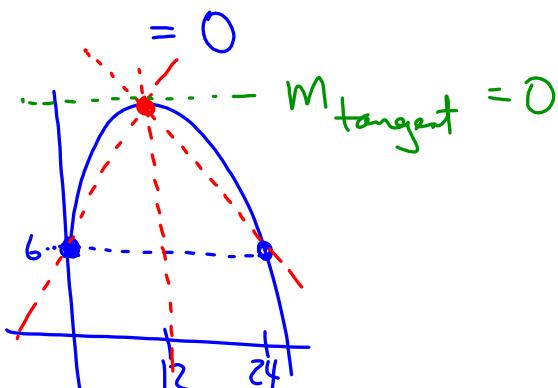
Sep 11-1:18 PM

$$7. P(t) = -1.5t^2 + 36t + 6$$

$$(a) \quad 2000 \quad 2024 \\ t=0 \quad t=24$$

$$\text{avg RoC} = \frac{P(24) - P(0)}{24 - 0}$$

⋮



Sep 11-1:20 PM

P.85 #10.  $V(r) = \frac{4}{3}\pi r^3$   
at  $r = 5\text{ cm}$

to estimate inst RoC :

- ① find avgRoC  
for each interval  
type  
(i) preceding  
(ii) central  
(iii) following  
then average  
them.

- ② series of  
decreasing  
interval widths,  
using one interval  
type, and look  
for the trend.  
 $\text{avgRoC} = \frac{f(a+h) - f(a)}{h}$

choose preceding  
try  $(5, V(5))$   
 $(49, V(49))$

$$\text{avg RoC} = \frac{V(5) - V(49)}{5 - 49} \\ \doteq \frac{523.6 - 492.8}{-0.1} \\ \doteq 307.9$$

use  $(5, V(5))$   
 $(4.99, V(4.99))$

$$\text{avg RoC} = \frac{V(5) - V(4.99)}{0.01} \\ \doteq 313.65$$

$$\text{avg RoC} = \frac{V(5) - V(4.999)}{0.001} \\ \doteq 314.1$$

Sep 12-1:59 PM