Dividing Polynomials

Sept 23/2014

Ex.1 What is $107 \div 4$?

recall: long division!

quotient

$$\begin{array}{r} 26 \\ 4\,\overline{)\,107} \\ \underline{8} \\ 27 \\ \underline{24} \\ 3 \end{array}$$

dividend

divisor

$3 \rightarrow$ remainder 3

$$107 \div 4 = 26 \text{ R } 3$$

$$= 26 + \frac{3}{4}$$

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Ex.2 Determine the quotient and remainder for

$$(3x^3 - 5x^2 - 7x - 1) \div (x - 3)$$

$$\begin{array}{r} 3x^2+4x+5 \\ x-3\,\overline{)\,3x^3-5x^2-7x-1} \\ \underline{-(3x^3-9x^2)} \\ 4x^2-7x \\ \underline{4x^2-12x} \\ 5x-1 \\ \underline{5x-15} \\ 14 \end{array}$$

① start with highest order term for divisor & dividend

$$\frac{3x^3}{x} = 3x^2$$

$$(3x^3-5x^2-7x-1) \div (x-3)$$

$$= 3x^2+4x+5 + \frac{14}{x-3}$$

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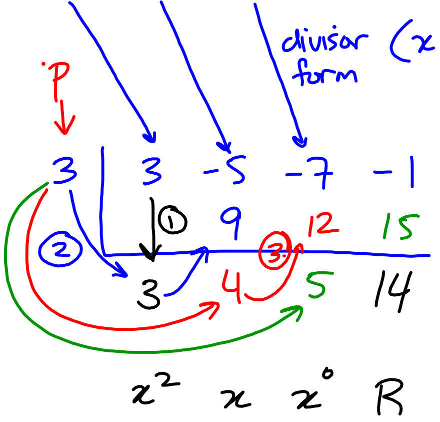
Ex.3 Use synthetic division (see p.164 for more detail)

$$(3x^3 - 5x^2 - 7x - 1) \div (x - 3)$$

divisor form $(x-p)$ $p=3$



① bring down leading coefficient

* assume divisor always $(1x-p)$

② multiply P by lead coefficient

③ add vertical columns of values

$$3x^2+4x+5+\frac{14}{x-3}$$

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Notes on synthetic division:

(1) The divisor must be in the form $(x-k)$

(2) All terms must be represented, even if they have a coefficient of zero

If the remainder of the division is zero, then both the quotient and the divisor are factors of the original polynomial.

Ex.4 Is $(x+2)$ a factor of $13x - 2x^3 + x^4 - 6$

p.168 # 1, 4, 5ace, 6ace, 7ac, 8d, 9ac, 10ace, 11, 12, 14

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p.168 # 1, 4, 5ace, 6ace, 7ac, 8d, 9ac, 10ace, 11, 12, 14

a e e ae

4.

$$\begin{array}{r} Q \\ \text{quotient} \\ \text{divisor}\,\overline{)\,\text{dividend}} \\ d \qquad D \end{array} \qquad \begin{array}{c}\text{remainder}\\ R\end{array}$$

$$\frac{D}{d} = Q + \frac{R}{d}$$

(a) want R

$$\frac{D}{d} - Q = \frac{R}{d} \quad [\times d]$$

$$D - Qd = R$$

$$R = (2x^3 - 5x^2 + 8x + 4) - (2x^2 - 11x + 41)(x+3)$$

$$= 2x^3 - 5x^2 + 8x + 4 - (2x^3 + 6x^2 - 11x^2 - 33x + 41x + 123)$$

$$\underbrace{}_{-5x^2} \quad \underbrace{}_{+8x}$$

$$= -119$$

(b) want D

$$\frac{D}{d} = Q + \frac{R}{d} \quad [\times d]$$

$$D = Qd + R$$

(c) d=?

$$\frac{D}{d} = Q + \frac{R}{d}$$

$$\frac{D}{d} - \frac{R}{d} = Q$$

$$\frac{1}{d}(D - R) = Q$$

$$D - R = Qd$$

$$\frac{D-R}{Q} = d$$

$$\frac{(6x^4 + 2x^3 + 3x^2 - 11x - 9) - (-5)}{2x^3 + x - 4} = d$$

$$\frac{6x^4 + 2x^3 + 3x^2 - 11x - 4}{2x^3 + x - 4} = d$$

$$\begin{array}{r} 3x + 1 \\ 2x^3 + 0x^2 + x - 4\,\overline{)\,6x^4 + 2x^3 + 3x^2 - 11x - 4} \\ \underline{6x^4 + 0x^3 + 3x^2 - 12x } \\ 2x^3 + 0x^2 + x - 4 \\ 2x^3 + 0x^2 + x - 4 \end{array}$$

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1

1 1

1 2 1

1 3 3 1

1 3 6 3 1

$$(a+b)(a+b)(a+b)$$

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5 (a)

$$x^2+4x+14$$

$$x-4\,\overline{)\,x^3+0x^2-2x+1}$$

$$\underline{x^3-4x^2}$$

$$4x^2-2x$$

$$\underline{4x^2-16x}$$

$$14x+1$$

$$\underline{14x-56}$$

$$57$$

(e)

$$x+1$$

$$x^3-x^2-x+1\,\overline{)\,x^4+0x^3+6x^2-8x+12}$$

$$\underline{x^4-x^3-x^2+x} \downarrow$$

$$x^3+7x^2-9x+12$$

$$\underline{x^3-x^2-x+1}$$

$$8x^2-8x+11 \rightarrow R$$

$$\frac{D}{d}=x+1+\frac{8x^2-8x+11}{x^3-x^2-x+1}$$

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6 (e)

$$6x^3-31x^2+45x-18$$

$$2x+1\,\overline{)\,12x^4-56x^3+59x^2+9x-18}$$

$$\underline{12x^4+6x^3} \downarrow$$

$$-62x^3+59x^2$$

$$\underline{-62x^3-31x^2}$$

$$90x^2+9x$$

$$\underline{90x^2+45x}$$

$$-36x-18$$

$$\underline{-36x-18}$$

$$0$$

$$\boxed{Q=6x^3-31x^2+45x-18}$$

$$(2x+1)=2(1x+\tfrac{1}{2})$$

$$\begin{array}{r|rrrrr} -\frac{1}{2} & 12 & -56 & 59 & 9 & -18 \\ & \downarrow & -6 & 31 & -45 & 18 \\ \hline & 12 & -62 & 90 & -36 & 0 \end{array}$$

$$Q=\frac{12x^3-62x^2+90x-36}{2}$$

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7(c)

$$\frac{\boxed{D}\cancel{d}}{\checkmark\cancel{d}} = \overset{\checkmark}{Q}d + \frac{\overset{\checkmark}{R}\cancel{d}}{\cancel{d}\checkmark} \qquad [\times d]$$

$$D = Qd + R$$

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8(d)

$$(x^2+1)(2x^3-1) + r = 2x^5+2x^3+x^2+1$$

$$2x^5 - x^2 + 2x^3 - 1 + r = 2x^5 + 2x^3 + x^2 + 1$$

$$r = 2x^2 + 2$$

$$\begin{array}{r} 2x^3 - 1 \\ x^2+1 \overline{\smash{\big)}\, 2x^5 + 2x^3 + x^2 + 1} \end{array}$$

R

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10 (a) $1x+5 \longrightarrow k=-5$

$$\begin{array}{c|cccc} \overset{k}{\downarrow} & & & & \\ -5 & 1 & 6 & -1 & -30 \\ & \downarrow & -5 & -5 & 30 \\ \hline & 1 & 1 & -6 & \underbrace{0}_{R=0} \end{array}$$

(e) $3x+5 = 3\left(x+\frac{5}{3}\right) \quad k=-\frac{5}{3}$

$\underline{\underline{OR}}$

$$x^5+0x^4+0x^3+0x^2+3x+\frac{2}{3}$$
$$3x+5\,\overline{)\,3x^6+5x^5+0x^4+0x^3+9x^2+17x-1}$$
$$\underline{3x^6+5x^5}$$
$$0x^5+0x^4$$
$$\underline{0x^5+0x^4}$$
$$0x^4+0x^3$$
$$\underline{0x^4+0x^3}$$
$$0x^3+9x^2$$
$$\underline{0x^3+0x^2}$$
$$9x^2+17x$$
$$\underline{9x^2+15x} \nearrow -\frac{3}{3}$$
$$2x-1$$
$$\underline{2x+\frac{10}{3}}$$
$$-\frac{13}{3}$$

$\therefore$ 3x+5 is $\underline{\text{NOT}}$ a factor

$$x^5+3x+\frac{2}{3}$$
$$\underbrace{3x+5}_{\text{2 terms}}\,\overline{)\,3x^6+5x^5+0x^4+0x^3+9x^2+17x-1}$$
$$\underline{3x^6+5x^5}$$
$$0 \quad 0 \quad 0 \quad \underbrace{9x^2+17x}_{\text{2terms}}$$
$$\underline{9x^2+15x}$$
$$2x-1$$

$$(\underset{?}{q})(3x)=2x$$
$$q=\frac{2x}{3x}$$
$$=\frac{2}{3}$$

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12. (a)

$$4x^2+3x\ \boxed{-5}$$
$$2x+1\,\overline{)\,8x^3+10x^2-px-5}$$
$$\underline{8x^3+4x^2}$$
$$6x^2-px$$
$$\underline{6x^2+3x}$$
$$-px-3x$$
$$(-p-3)x-5$$
$$\underline{\boxed{-10x}\ \boxed{-5}}$$
$$0+0 \text{ remainder}$$

$$\frac{(-p-3)x}{x}-\frac{(-10x)}{x}=\frac{0}{x}$$

$$-p-3+10=0$$
$$-p+7=0$$
$$7=p$$

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