Dividing Polynomials

Sept 23/2014

Ex.1 What is $107 \div 4$?
recall: long division!

$$\begin{array}{r} 26 \\ 4\overline{)107} \\ -8\downarrow \\ \hline 27 \\ 24 \\ \hline \textcircled{3} \end{array}$$

→ remainder of 3 after division by 4

$$4\overline{)107}$$

$$107 \div 4 = 26 R3$$

$$= 26 + \frac{3}{4}$$

Sep 22-7:00 PM

Ex.2 Determine the quotient and remainder for

$$(3x^3 - 5x^2 - 7x - 1) \div (x - 3)$$

$$\begin{array}{rrrrr} & 3x^2 & +4x & +5 & \\ x-3\,) & 3x^3 & -5x^2 & -7x & -1 \\ \overset{?}{-}(& 3x^3 & -9x^2) & \downarrow & \downarrow \\ \hline & & 4x^2 & -7x & \\ & -(& 4x^2 & -12x) & \\ \hline & & & 5x & -1 \\ & & & 5x & -15 \\ \hline & & & & 14 \rightarrow R14 \end{array}$$

① focus on highest order terms

$$\therefore (3x^3-5x^2-7x-1)\div(x-3)$$

$$= 3x^2+4x+5 + \frac{14}{x-3}$$

Sep 22-7:12 PM

Ex.3 Use synthetic division (see p.164 for more detail)

$$(3x^3-5x^2-7x-1) \div \underbrace{(x-3)}$$

$x-p,\ p=3$

$$\begin{array}{c|cccc} p & & & & \\ 3 & 3 & -5 & -7 & -1 \\ & \downarrow & 9 & 12 & 15 \\ \hline & 3 & 4 & 5 & 14 \\ & & & & R \end{array}$$

$\leftarrow$ add (instead of subtract from long division)

$$\underbrace{3x^2}_{} +4x+5+\frac{14}{x-3}$$

first division was $\frac{3x^3}{x}=3x^2$

$$f(x) \div \left[2(x-3)\right]$$
$$= \frac{f(x)}{2} \div (x-3)$$
$$= \frac{1}{2}f(x) \div (x-3)$$

Sep 22-7:12 PM

Notes on synthetic division:

(1) The divisor must be in the form $(x-k)$

(2) All terms must be represented, even if they have a coefficient of zero

If the remainder of the division is zero, then both the quotient and the divisor are <u>factors</u> of the original polynomial.

Ex.4 Is $(x+2)$ a factor of $13x-2x^3+x^4-6$?

Sep 22-7:31 PM

p.168 # 1, 4, 5ace, 6ace, 7ac, 8d, 9ac, 10ace, 11, 12, 14

(ii) b

1.(a) (ii) $x+4$

$$\begin{array}{rl}
 & x^3-20x^2+84x-326 \\ \cline{2-2}
x+4 \,\big) & x^4-16x^3+4x^2+10x-11 \\
 & \underline{x^4+4x^3} \\
 & \quad -20x^3+4x^2 \\
 & \quad \underline{-20x^3-80x^2} \\
 & \qquad\quad 84x^2+10x \\
 & \qquad\quad \underline{84x^2+336x} \\
 & \qquad\qquad -326x-11 \\
 & \qquad\qquad \underline{-326x-1304} \\
 & \qquad\qquad\qquad 1293
\end{array}$$

$$x^3-20x^2+84x-326+\frac{1293}{x+4}$$

Sep 22-7:46 PM

1 (a) (ii)

$1x+4 \rightarrow k=-4$

$$\begin{array}{r|rrrrr}
-4 & 1 & -16 & 4 & 10 & -11 \\
 & \downarrow & -4 & 80 & -336 & 1304 \\ \hline
 & 1 & -20 & 84 & -326 & 1293
\end{array}$$

$$\underbrace{1\quad -20\quad 84\quad -326}_{\text{quotient}}\quad \underbrace{1293}_{\text{remainder}}$$

Sep 24-9:20 AM

4. $$\begin{array}{r} \text{quotient} \\ \text{divisor}\overline{)\ \text{dividend}} \end{array}$$

R

$$\text{dividend} \div \text{divisor} = \text{quotient} + \frac{R}{\text{divisor}}$$

$$\frac{\text{dividend}}{\text{divisor}} = \text{quotient} + \frac{R}{\text{divisor}}$$

(a) $$\frac{\text{dividend}✓}{\text{divisor}✓} = \text{quotient}✓ + \frac{R?}{\text{divisor}✓}$$

✓ ① division → find R, find quotient

ok ② rearrange

$$\frac{\text{dividend}}{\text{divisor}} - \text{quotient} = \frac{R}{\text{divisor}}$$

$$\text{dividend} - (\text{quotient})(\text{divisor}) = R$$

(b) dividend?

$$\frac{D}{d} = Q + \frac{R}{d}$$

$$D = Qd + R$$

$$D = (3x^3-5x+8)(2x+4)-3$$

$$= 6x^4+12x^3-10x^2-20x +16x+32-3$$

$$= 6x^4+12x^3-10x^2-4x+29$$

(c) $$\frac{D}{d?} = Q + \frac{R}{d}$$

$$\frac{6x^4+2x^3+3x^2-11x-9}{A} = 2x^3+x-4+\frac{-5}{A}$$

$$6x^4+2x^3+3x^2-11x-9 = (2x^3+x-4)A-5$$

$$6x^4+2x^3+3x^2-11x-4 = (2x^3+x-4)A$$

$$\frac{6x^4+2x^3+3x^2-11x-4}{2x^3+x-4} = A$$

$$\begin{array}{r} 3x+1 \\ 2x^3+0x^2+x-4\overline{)6x^4+2x^3+3x^2-11x-4} \\ \underline{6x^4+0+3x^2-12x}\ \downarrow \\ 2x^3+0x^2+x-4 \\ \underline{2x^3+0x^2+x-4} \\ 0 \end{array}$$

$\therefore A = 3x+1$

Sep 24-9:22 AM

8 (d)

$$(x^2+1)(2x^3-1)+R = 2x^5+2x^3+x^2+1$$

$$\frac{D}{d} = Q + \frac{R}{d}$$

$$D✓ = Q✓d✓ + R$$

① long way

$$\begin{array}{r} 2x^3-1 \\ x^2+1\overline{)\ 2x^5+2x^3+x^2+1} \\ \underline{\qquad\qquad} \\ \underline{\qquad\qquad} \\ \underline{\qquad\qquad} \\ \underline{\qquad\qquad} \end{array}$$

R = ____

② short way:

$$(x^2+1)(2x^3-1)+R = 2x^5+2x^3+x^2+1$$

$$\cancel{2x^5}-x^2+\cancel{2x^3}-1+R = \cancel{2x^5}+\cancel{2x^3}+x^2+1$$

$$R = 2x^2+2$$

Sep 25-10:28 AM

5(e)

$$\begin{array}{r l}
 & x+1 \\
x^3-x^2-x+1 \,\big) & \overline{x^4+0x^3+6x^2-8x+12} \\
 & \underline{x^4-x^3-x^2+x} \quad \downarrow \\
 & \quad x^3+7x^2-9x+12 \\
 & \quad \underline{x^3-x^2-x+1} \\
 & \quad \underbrace{8x^2-8x+11}_{R}
\end{array}$$

$$\frac{x^4+6x^2-8x+12}{x^3-x^2-x+1} = x+1+\frac{8x^2-8x+11}{x^3-x^2-x+1}$$

Sep 24-9:42 AM

10(e) $3x+5 \rightarrow 3(1x+\frac{5}{3}) \rightarrow k=\frac{-5}{3}$

$3x+5 \,\big)\overline{\qquad\qquad\qquad}$

$$\begin{array}{r|rrrrrrl}
-\frac{5}{3} & 3 & 5 & 0 & 0 & 9 & 17 & -1 \nearrow = \frac{-3}{3} \\
 & \downarrow & -5 & 0 & 0 & 0 & -15 & -\frac{10}{3} \\
\hline
 & 3 & 0 & 0 & 0 & 9 & 2 & \boxed{-\frac{13}{3}}
\end{array}$$

$R \neq 0$ ∴ not a factor $\qquad R = \frac{-13}{3}$

Sep 25-10:34 AM

11. $V =$ ________

$\ell = (x+3)$ $w = (x+2)$ $h = ?$

$$V = \ell \times w \times h$$

$$h = \frac{V}{\ell \times w}$$

$$h = \frac{V}{(x+3)(x+2)}$$

① $$x+3 \overset{Q}{\overline{\big)\; V}} \qquad x+2 \overset{h}{\overline{\big)\; Q}}$$

② $$(x+2)(x+3) \overline{\big)\; V}$$

$$\begin{array}{r}
x+1 \\
x^2+5x+6 \overline{\big)\, x^3+6x^2+11x+6} \\
\underline{x^3+5x^2+6x} \\
x^2+5x+6 \\
\underline{x^2+5x+6} \\
0
\end{array}$$

$$V = \underset{h}{(x+1)}\underset{w}{(x+2)}\underset{\ell}{(x+3)}$$

Sep 25-10:39 AM

14. $f(x) = x^n - 1$

$$\begin{array}{r}
x^{n-1}+x^{n-2}+x^{n-3}+\ldots+x^1+1 \\
x-1 \overline{\big)\, x^n+0x^{n-1}+\ldots+0x-1} \\
\underline{x^n-x^{n-1}} \\
x^{n-1}+0 \\
\underline{x^{n-1}-x^{n-2}} \\
x^{n-2} \\
x^2+0 \\
\underline{x^2-x} \\
x-1 \\
\underline{x-1} \\
0
\end{array}$$

$$x^2-1^2 = (x-1)(x+1)$$

$$x^3-1^3 = (x-1)(x^2+x+1)$$

$$x^4-1^4 = (x^2)^2-(1^2)^2$$

$$= (x^2-1)(x^2+1)$$

$$= (x-1)(x+1)(x^2+1)$$

Sep 25-10:43 AM