

Factoring Polynomials

Sept 25/2014

Remainder Theorem: When a polynomial, $f(x)$, is divided by $x - a$, the remainder is equal to $f(a)$.

Factor Theorem:

If the remainder, or $f(a)$, is equal to zero, then $x - a$ is a factor of the polynomial $f(x)$.

Ex.1 Use the factor theorem to determine one factor of

$$f(x) = x^3 + 4x^2 + x - 6$$

then completely factor the function.

$$\frac{f(x)}{x-a} = q(x)$$

$$f(x) = (x-a) q(x)$$

$$f(a) = 0 \quad q(a)$$

Sep 23-9:24 AM

Ex.1 Use the factor theorem to determine one factor of
 $f(x) = x^3 + 4x^2 + x - 6$
 then completely factor the function.

guess + check values for a

$$\text{try } a=0: f(0) = (0)^3 + 4(0)^2 + (0) - 6 \\ = -6$$

$$\text{try } a=1: f(1) = (1)^3 + 4(1)^2 + (1) - 6 \\ = 1 + 4 + 1 - 6 \\ = 0$$

$\therefore (x-1)$ is a factor.

① divide by $(x-1)$

$$\begin{array}{r} x^2 + 5x + 6 \\ \hline x-1) x^3 + 4x^2 + x - 6 \\ x^3 - x^2 \\ \hline 5x^2 + x \\ 5x^2 - 5x \\ \hline 6x - 6 \\ 6x - 6 \\ \hline 0 \end{array}$$

② keep trying factor theorem

* not recommended
 → there might not be another factor

$$\begin{aligned} x^3 + 4x^2 + x - 6 &= (x-1)(x^2 + 5x + 6) \\ &= (x-1)(x+2)(x+3) \end{aligned}$$

Sep 23-6:05 PM

A rational number can be expressed as a fraction with an integer numerator and denominator (but no division by zero).

$$\frac{a}{b} \text{ where } a, b \in \mathbb{Z}, b \neq 0$$

*↑
integers*

A rational root is a zero which is a rational number. For a polynomial, roots can be expressed as factors:

$$\left(x - \frac{a}{b} \right), \text{ or, more commonly, } (bx - a)$$

Sep 23-9:32 AM

The rational roots test allows us to limit our search for roots using the leading term and the constant (last) term.

For a polynomial in the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$$

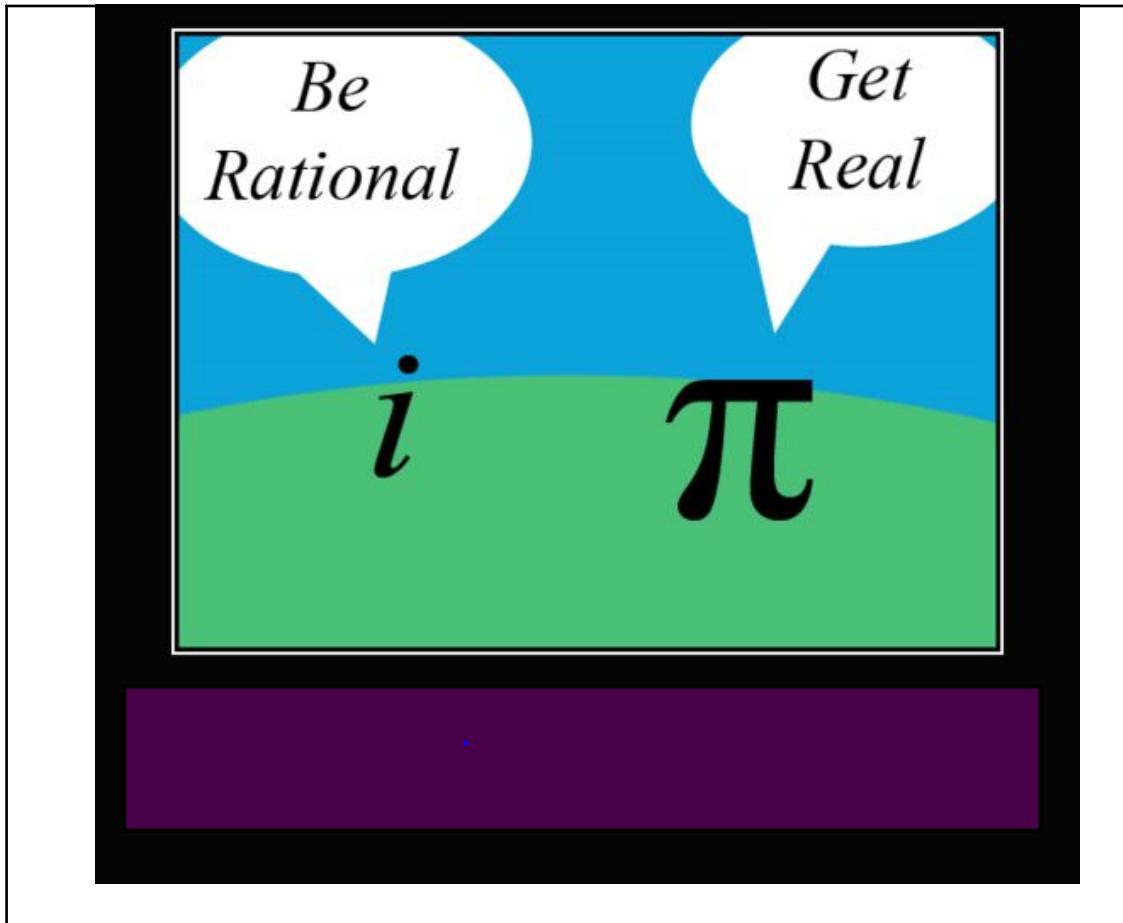
the possible rational roots are

$$\frac{\text{all factors of constant term}}{\text{all factors of leading coefficient}}$$

note: Some roots are irrational, and there is no guarantee that the rational root test will be successful.

Therefore, not all polynomials will be factorable.

Sep 23-9:44 AM



Sep 23-6:14 PM

For example, $y = 3x^2 + 10x - 8$ has a constant term -8 and a leading coefficient of 3.

factors of 8 are 1, 2, 4, 8

factors of 3 are 1, 3

$$\text{possible rational roots are } \frac{\pm 1, 2, 4, 8}{1, 3}$$

As a list: $\pm 1, \pm 2, \pm 4, \pm 8, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}, \pm \frac{8}{3}$

Using the factor theorem, we can test each one of these until $f(a) = 0$. For this quadratic, $f(-4) = 0$ $f\left(\frac{2}{3}\right) = 0$

$$\begin{aligned} 3x^2 + 10x - 8 &= (x + 4)(3)\left(x - \frac{2}{3}\right) \\ &= (x + 4)(3x - 2) \end{aligned}$$

Sep 23-5:38 PM

Ex.2 Determine all possible rational roots for

$$f(x) = 2x^3 + 7x^2 - 64x - 105$$

then show that $2x+3$ is a factor.

$$105 : 1, 3, 5, 7, 15, 21, 35, 105$$

$$2 : 1, 2$$

$$\begin{array}{r} \text{possible} \\ \text{rational} \\ \text{roots} \end{array} \quad \begin{array}{c} \pm 1, 3, 5, 7, 15, 21, 35, 105 \\ \hline 1, 2 \end{array}$$

$$\pm 1, 3, 5, 7, 15, 21, 35, 105, \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \frac{15}{2}, \frac{21}{2}, \frac{35}{2}, \frac{105}{2}$$

$$\text{Show } 2x+3 = 2\left(x + \frac{3}{2}\right)$$

$$\begin{aligned} f\left(-\frac{3}{2}\right) &= 2\left(-\frac{3}{2}\right)^3 + 7\left(-\frac{3}{2}\right)^2 - 64\left(-\frac{3}{2}\right) - 105 \\ &\vdots \\ &= 0 \end{aligned}$$

Sep 23-6:06 PM

Assigned Work:

p.176 # [1 - 3][basics],
4bf, 5ac, 6abc, 7df, 8, 9, 10, 13, 17
c f

$$6(c) \quad f(x) = x^4 + 8x^3 + 4x^2 - 48x$$

$$= x \underbrace{(x^3 + 8x^2 + 4x - 48)}$$

$$\text{Possible factors} = \frac{\pm 1, 2, 3, 4, 6, 8, 12, 16, 24, 48}{1}$$

$$f(1) = (1)(1+8+4-48) \neq 0$$

$$f(2) = (2)(8+32+8-48) \neq 0$$

$x-2$ is a factor

$$\begin{array}{r} 2 | 1 \ 8 \ 4 \ -48 \\ \quad \quad 2 \ 20 \ 48 \\ \hline 1 \ 10 \ 24 \ 0 \\ x^2 \ x^1 \ x^0 \ R \end{array}$$

$$\begin{aligned} f(x) &= x(x-2)(x^2 + 10x + 24) \quad \begin{array}{r} S \ 10 \\ P \ 24 \\ I \ 6, 4 \end{array} \\ &= x(x-2)(x+6)(x+4) \end{aligned}$$

Sep 23-6:17 PM

$$f(x) = x^5 - x^4 + 2x^3 - 2x^2 + x - 1$$

$$\text{possible factors} = \frac{\pm 1}{1}$$

$$f(1) = 1 - 1 + 2 - 2 + 1 - 1 \\ = 0$$

$x-1$ is a factor.

$$\begin{array}{r} | \begin{array}{cccccc} 1 & -1 & 2 & -2 & 1 & -1 \\ | & 1 & 0 & 2 & 0 & 1 \end{array} \\ \hline \begin{array}{cccccc} 1 & 0 & 2 & 0 & 1 & 0 \\ x^4 & x^3 & x^2 & x^1 & x^0 & R \end{array} \end{array}$$

$$f(x) = (x-1)(x^4 + 2x^2 + 1)$$

$$x^2 + 2x + 1 = (x+1)^2$$

$$= (x-1)(x^2+1)^2$$

Sep 29-12:43 PM

$$9. \quad 12x^3 + kx^2 - x - 6, \quad (2x-1)$$

$$f\left(\frac{1}{2}\right) = 0 \quad \leftarrow \quad R=0$$

$$\left\{ 12 \left(\frac{1}{2}\right)^3 + k \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right) - b = 0 \right\} \text{ is a factor}$$

$$\frac{12}{8} + \frac{k}{4} - \frac{1}{2} - 6 = 0 \quad \begin{cases} \text{zero at } x = \frac{1}{2} \\ f(1) \end{cases}$$

$$12 + 2k - 4 - 48 = 0$$

$$2k = 40$$

$$k = 20$$

Sep 29-12:49 PM

$$10. \quad (x-1) \rightarrow R=10 \quad f(1)=10$$

$$x-2 \rightarrow R=51 \quad f(2)=51$$

$$f(1) = a - 1 + 2 + b$$

$$10 = a + 1 + b$$

$$9 = a + b \quad (1)$$

$$f(2) = a(2)^3 - (2)^2 + k(2) + b$$

$$51 = 8a - 4 + 4 + b$$

$$51 = 8a + b \quad (2)$$

Sep 29-12:55 PM

13

$$\left. \begin{array}{l} f(x) \div (x+2) \rightarrow R_1 \\ f(x) \div (x-2) \rightarrow R_2 \end{array} \right\} R_1 = 2R_2$$

$$R_1 = f(-2)$$

$$= (-2)^3 + 4(-2)^2 + k(-2) - 4$$

$$= -8 + 16 - 2k - 4$$

$$R_1 = -2k + 4$$

$$R_2 = f(2)$$

$$= (2)^3 + 4(2)^2 + k(2) - 4$$

$$= 8 + 16 + 2k - 4$$

$$R_2 = 2k + 20$$

$$R_1 = 2R_2$$

$$-2k + 4 = 2(2k + 20)$$

$$-2k + 4 = 4k + 40$$

$$-36 = 6k$$

$$k = -6$$

Sep 29-12:58 PM