Transformations of Polynomials

Sept 29/2014

Recall: Any function, $f(x)$, may be transformed to

$$y = a\,f\left[k\left(x-p\right)\right]+q$$

This is most easily accomplished by transforming key points or features of the parent function.

$$(x, y) \rightarrow \left(\frac{x}{k} + p, ay + q\right)$$

Cubic: $f(x) = x^3$

x	$f(x)$
-2	-8
-1	-1
0	0
1	1
2	8

Quartic: $f(x) = x^4$

x	$f(x)$
-2	16
-1	1
0	0
1	1
2	16

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When determining an equation from a transformed graph, there are some functions where the roles of 'a' and 'k' are redundant (i.e., you only need one of them, not both).

For polynomials, this will apply to any parent function in the form

$$f(x) = x^n$$

or any transformations of this function

$$y = a\left[k\left(x-p\right)\right]^n + q$$

x-y notation

With some graphs, it may be more convenient to work with 'a', while for others it may be 'k'. You can also choose to work with both. Be prepared to express your final answer in terms of 'a' only.

Note: The point (0,0) is only transformed by 'p' and 'q', so use this point (if available) to quickly determine 'p' and 'q'.

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$$y = (3x)^2 + 5$$

h. compression by 3

$$= 9(x)^2 + 5$$

$$= 9x^2 + 5$$

v. stretch by 9

$$y = (2x)^3$$
$$= 8x^3$$

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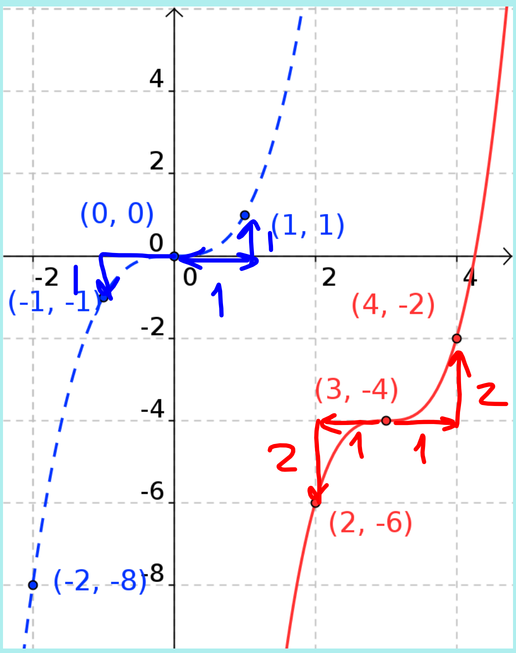
Ex. Determine equation of cubic, best to use 'a'
(horizontal spacing of 'good' points matches parent)



$(0,0) \rightarrow (3,-4)$
p q

$|a| = 2$

$a > 0$ (end behaviour)

$$y = 2(x-3)^3 - 4$$

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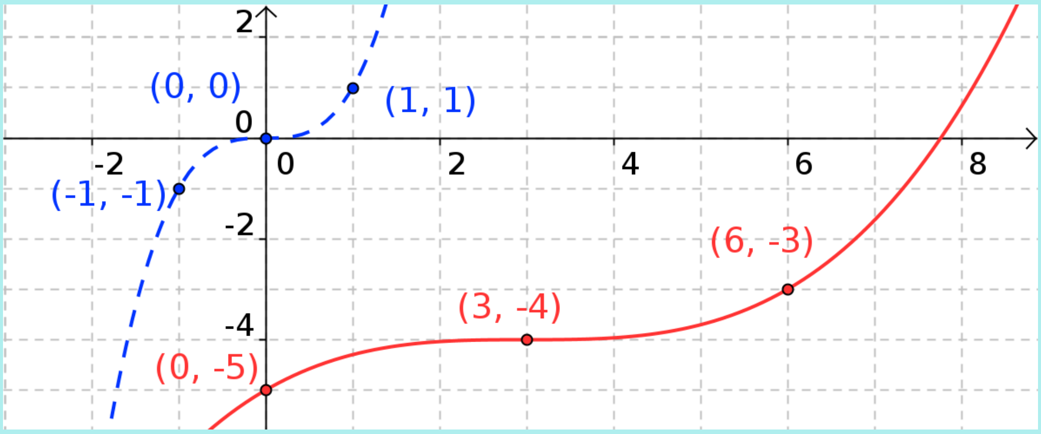
Ex. Determine equation of cubic, best to use 'k'
(vertical spacing of 'good' points matches parent)



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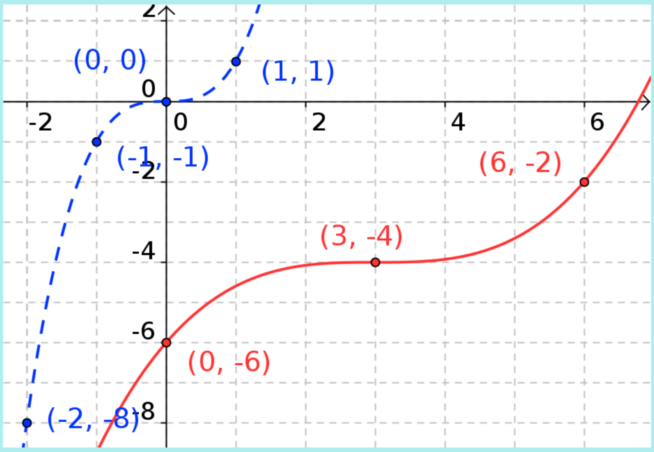
Ex. Determine equation of cubic, using 'a' and 'k'
(no matching between horizontal or vertical spacing)



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Assigned Work:

p.155 # 2, 3, 4bde, 5, 6bde, 7, 8, 9, 10ace, 11, 14

e d ff

2(e) $y = -4.8(x-3)(x-3)$

$= -4.8(x-3)^2$ $\qquad f(x) = x^2$

4(d) $y = (x+9)^3$

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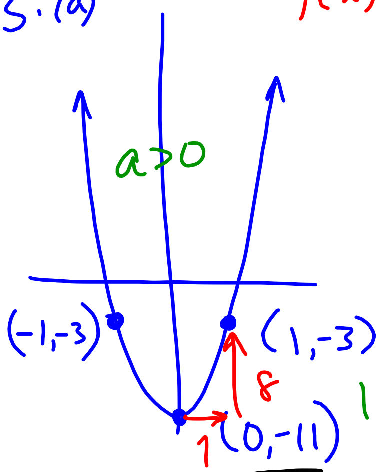
5.(a) $\qquad f(x) = x^n$



Δx	x	$y = x^2$
	-2	4
1	-1	1
1	0	0
1	1	1
1	2	2

$|a| = 8$

$k = 1$

$p = 0, \; q = -11$

$y = 8(x-0)^2 - 11$

$= 8x^2 - 11$

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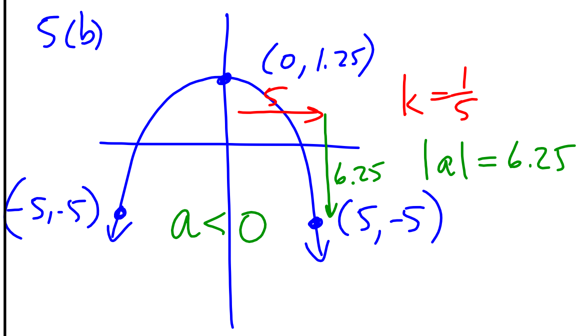
5(b)



$p=0,\ q=1.25$

$$y = -6.25\left[\tfrac{1}{5}(x-0)\right]^2 + 1.25$$

$$= -6.25\left(\tfrac{1}{5}x\right)^2 + 1.25 \checkmark$$

$$= -6.25\left(\tfrac{1}{5}\right)^2 x^2 + 1.25 \qquad (ab)^x = a^x b^x$$

$$= \frac{-6.25}{25}x^2 + 1.25$$

$$= -0.25x^2 + 1.25 \checkmark$$

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6(d) $f(x) = x^3$

$(-1,-1),\ (0,0),\ (2,8)$

$a = \frac{1}{10} \quad k = \frac{1}{7} \quad q = -2$

$$(x,y) \rightarrow \left(\frac{x}{k}+p,\ ay+q\right)$$

$$\rightarrow \left(\frac{x}{\frac{1}{7}}+0,\ \frac{1}{10}y-2\right)$$

$$\rightarrow \left(7x,\ \frac{y}{10}-2\right)$$

$$(2,8) \rightarrow \left(7(2),\ \frac{8}{10}-2\right)$$

$$\rightarrow \left(14,\ \frac{4}{5}-\frac{10}{5}\right)$$

$$\rightarrow \left(14,\ \frac{-6}{5}\right)$$

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8. ① v. reflect

② $a = -\frac{2}{3}$

③ $p = 13$

④ $q = -13$

$k = 1$

$(x,y) \to \left(\frac{x}{k} + p,\ ay + q\right)$

$\to \left(x + 13,\ -\frac{2}{3}y - 13\right)$

$(x,y) \to \left(11,\ -\frac{23}{3}\right)$
?

$x + 13 = 11$
$x = -2$

$-\frac{2}{3}y - 13 = -\frac{23}{3}$ $[\times 3]$
$-2y - 39 = -23$
$-2y = 16$
$y = -8$

$\therefore (-2,-8) \to \left(11, -\frac{23}{3}\right)$

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$y = -(2x+5)^3 - 20$

set $y = 0$

$0 = -(2x+5)^3 - 20$

$20 = -(2x+5)^3$

$-20 = (2x+5)^3$

$\sqrt[3]{-20} = 2x + 5$

$\sqrt[3]{-20} - 5 = 2x$

$x = \frac{\sqrt[3]{-20} - 5}{2}$

$\doteq$ ________

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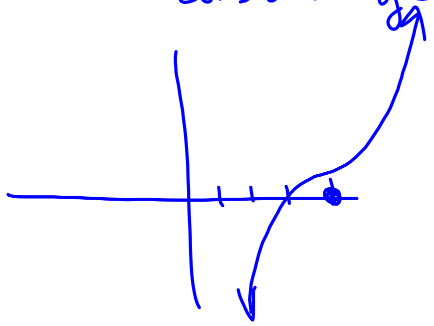
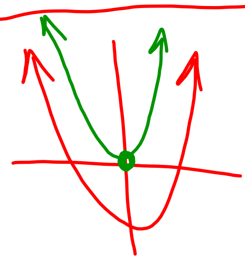
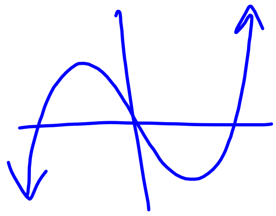
10(a) $y = 2(x-4)^3 + 1$

1 zero. $\longrightarrow$ solve for $y=0$

or consider $y = 2(x-4)^3$







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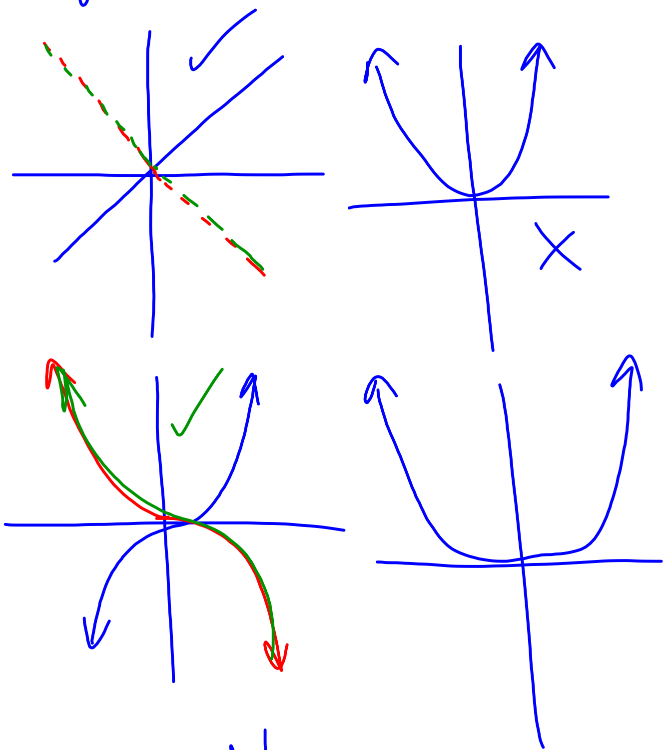
11. $y = x^n$



n is odd!

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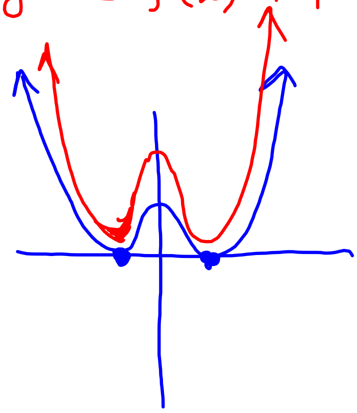
14. $y = \underbrace{(x-1)^2(x+1)^2}$

to

$y = 2\underbrace{(x-1)^2(x+1)^2}_{f(x)}+1$

$y = 2\ f(x) + 1$



no zeroes

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