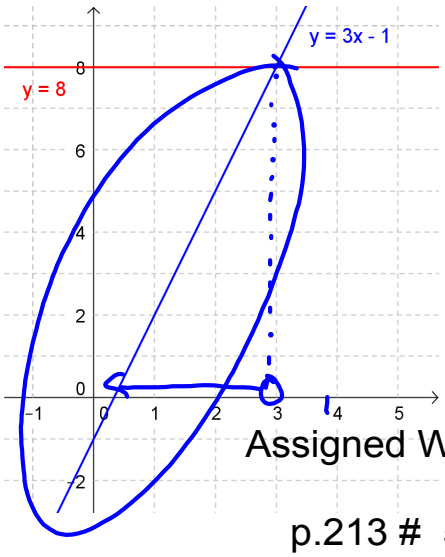
Solving Linear Inequalities

To solve an inequality, find all values that satisfy the inequality.

Consider: $3x - 1 < 8$

The simplest way to visualize the solution is to graph and compare the LS and RS:



Where is the line y = 3x - 1 less than the line y = 8?

$x < 3$

Assigned Work:

p.213 # 5bdf, 6be, 7bdf, 8, 9, 11, 15, 19

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We have also solved such inequalities by: $3x - 1 < 8$

(1) solving the corresponding equation, then
(2) testing values around the solution(s).

(1) Solve $3x - 1 = 8$

$3x = 8 + 1$
$3x = 9$
$x = 3$

(2) Test x < 3 and x > 3

LS
$= 3(2) - 1$
$= 7$
< 8
∴ passes

LS $= 3(4) - 1$
$= 12 - 1$
$= 11$
> 8
∴ fails

∴ $x < 3$

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$3x - 1 < 8$

(1) Solve $3x - 1 = 8$

$3x = 9$

$x = 3$

(2) Test $x < 3: \; 3(2) - 1 = 5$, pass

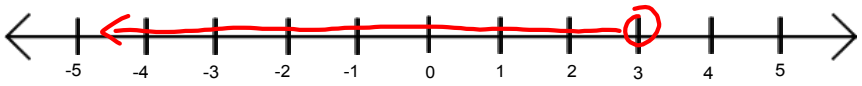
Test $x > 3: \; 3(4) - 1 = 11$, fail

The solution can be represented as:

(a) set notation: $\{x \in \mathbb{R} | x < 3\}$

(b) interval notation: $x \in (-\infty, 3)$

(c) a number line:



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Algebraic Operations on Inequalities

What are the effects of adding, subtracting, multiplying, and dividing on a very simple inequality?

Start with $4 < 8$, which is obviously true.

add positive: $4 + 2 < 8 + 2$, $6 < 10$ ✓

add negative: $4 + (-2) < 8 + (-2)$, $2 < 6$ ✓

subtract positive: $4 - 2 < 8 - 2$, $2 < 6$ ✓

subtract negative: $4 - (-2) < 8 - (-2)$, $6 < 10$ ✓

multply by positive: $4 \times 2 < 8 \times 2$, $8 < 16$ ✓

multiply by negative: $4 \times (-2) < 8 \times (-2)$, $-8 < -16$ ✗

divide by positive: $\frac{4}{2} < \frac{8}{2}$, $2 < 4$ ✓

divide by negative: $\frac{4}{-2} < \frac{8}{-2}$, $-2 < -4$ ✗

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Solving Inequalities Algebraically:

We can use the same basic operations (add, subtract, multiply, divide) that we would with a regular equation.

Note: When multiplying or dividing by a negative value, the direction of the inequality must be switched.

eg $3x - 5 > 10$

$3x > 10 + 5$

$3x > 15$

$x > 5$

eg $-3x - 5 > 10$

$-3x > 10 + 5$

$-3x > 15$

$x < -5$

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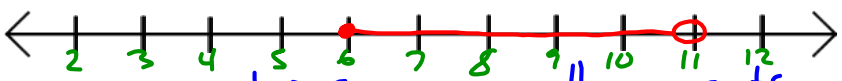
Ex.1 Solve $10 \leq 3(2x - 5) - (3x - 7) < 25$.

Express your solution using:
(a) set notation,
(b) interval notation,
(c) a number line.



- use opperations on all parts together
- simplify before starting

$10 \leq 6x - 15 - 3x + 7 < 25$

$10 \leq 3x - 8 < 25$

$10 + 8 \leq 3x < 25 + 8$

$\frac{18}{3} \leq \frac{3x}{3} < \frac{33}{3}$

$6 \leq x < 11$

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Assigned Work:

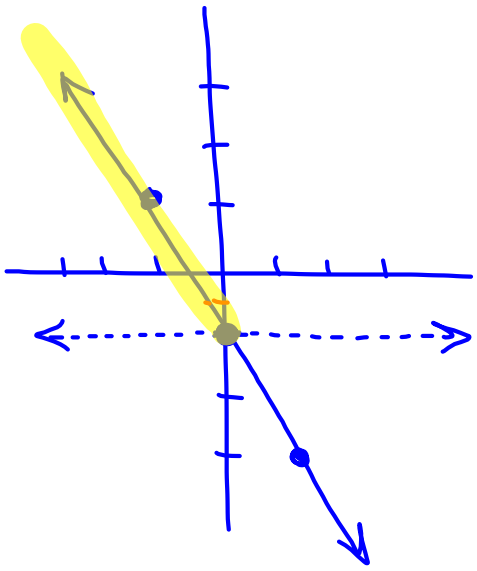
p.213 # 5bdf, 6be, 7bdf, 8, 9, 11, 15, 19

5. (b) $-2x-1 > -1$

$+1 \quad +1$

$\frac{-2x}{-2} > \frac{0}{-2}$

$x < 0$



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6. (b) $-6x < x+4 < 12$

does 0 make inequality true?

test $x=0$

$-6(0) < 0+4 < 12$?

$0 < 4 < 12$?

true true

$\therefore$ 0 is in solution set

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8. (a) $x > 4$ $[\times 3]$

$3x > 12$ $[-5]$

$3x - 5 > 7$ $[+x]$

$4x - 5 > x + 7$

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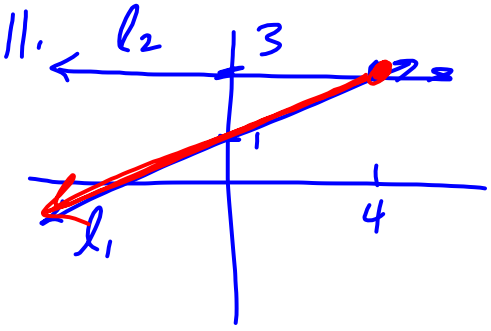
11.



$l_1 : y = \frac{1}{2}x + 1$ $l_2 : y = 3$

$\frac{1}{2}x + 1 \leq 3$

$\frac{1}{2}x \leq 2$

$x \leq 4$

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15. $|2x-1| < 7$

$-7 < 2x-1 < 7$

$-6 < 2x < 8$

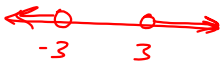
$-3 < x < 4$

$y = |2x-1|$

$= |2(x-\frac{1}{2})|$

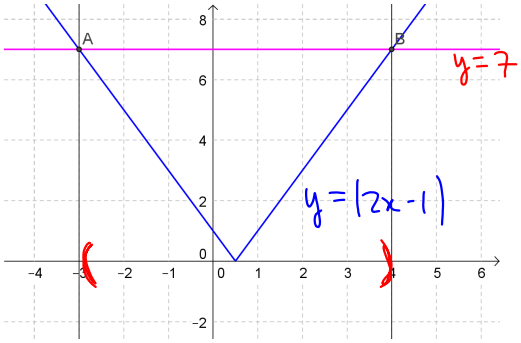
recall

$|x| > 3$



$|x| < 3$

$-3 < x < 3$



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