Unit 3: Polynomial & Rational Equations & Inequalities

Solving Polynomial Equations

Oct 1/2014

Recall: To solve an equation means finding the real roots of the equation.

When solving a quadratic equation, there are several options, such as:

- factoring to find the zeroes (roots)
- graphing
- completing the square (vertex form) and solving for y=0
- quadratic formula

Polynomial equations of degree 3 or higher can be solved by:

- graphing
- factoring down to degree 2 (quadratic), then applying one of the techniques listed above

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(1) Rewrite the equation so it is equal to zero.
(2) Define the resulting polynomial as a function and apply the factor theorem.
(3) Factor out the first term (polynomial division), and repeat until in a fully factored form.
(4) Find the roots of the equation (i.e., set it back to zero and solve).
(5) Ignore solutions that are outside of the domain defined by the conditions of the problem.

Ex.1 Solve $3x^3 + 8x^2 = -3x + 2$

① $3x^3+8x^2+3x-2=0$

② let $P(x)=3x^3+8x^2+3x-2$

possible roots $= \frac{\pm 1, 2}{1, 3}$ $\quad \pm 1, 2, \frac{1}{3}, \frac{2}{3}$

$P(-1) = 3(-1)^3+8(-1)^2+3(-1)-2$
$= -3+8-3-2$
$= 0$

$x+1$ is a factor

③

$$\begin{array}{r|rrrr} -1 & 3 & 8 & 3 & -2 \\ & \downarrow & -3 & -5 & 2 \\ \hline & 3 & 5 & -2 & \underbrace{0}_{R} \end{array}$$

$P(x) = (x+1)(3x^2+5x-2)$

$3x^2+5x-2$
$= 3x^2+6x-x-2$
$= 3x(x+2)-1(x+2)$
$= (x+2)(3x-1)$

S 5
P −6
F 6, −1

$P(x) = (x+1)(x+2)(3x-1)$

④ Set $P(x)=0$
$(x+1)(x+2)(3x-1)=0$

$x=-1,\ x=-2,\ x=\frac{1}{3}$

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Ex.2 Determine the exact roots of $x^3 - 4x^2 + 2x + 3$

let $P(x) = x^3 - 4x^2 + 2x + 3$

roots? $\pm \frac{1, 3}{1} \longrightarrow \pm 1, 3$

$P(1) = 2$ $\quad P(3) = 0$ $\quad x-3$ is a factor

$P(-1) = -4$

$$\begin{array}{r}
x^2 - x - 1 \\
x-3 \overline{) x^3 - 4x^2 + 2x + 3} \\
\underline{x^3 - 3x^2 } \\
-x^2 + 2x \\
\underline{-x^2 + 3x } \\
-x + 3 \\
\underline{-x + 3} \\
0 \checkmark
\end{array}$$

$P(x) = (x-3)\underbrace{(x^2 - x - 1)}_{\text{no rational roots}}$

consider $x^2 - x - 1 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{1 \pm \sqrt{1 - 4(1)(-1)}}{2(1)}$$

$$= \frac{1 \pm \sqrt{5}}{2}$$

$\therefore$ solutions (roots) are $3, \frac{1+\sqrt{5}}{2}, \frac{1-\sqrt{5}}{2}$

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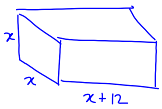
Ex.3 A box is in the shape of a rectangular prism. One side is a square, and the length is 12 units longer than the square sides. The volume of the box is 135 cubic units. What are the dimensions of the box?



$V = (x)(x)(x+12)$ $\quad \cancel{x^2 = 135}$

$135 = x^2(x+12)$ DO NOT $\longrightarrow$ $\cancel{x+12 = 135}$

$0 = x^2(x+12) - 135$

$0 = x^3 + 12x^2 - 135$

$P(x) = x^3 + 12x^2 - 135$

$P(3) = 0$ $\quad x-3$ is a factor

$$\begin{array}{r|rrrr}
3 & 1 & 12 & 0 & -135 \\
 & \downarrow & 3 & 45 & 135 \\
\hline
 & 1 & 15 & 45 & \underbrace{0}_{R}
\end{array}$$

$P(x) = (x-3)(x^2 + 15x + 45)$

cannot be factored

$D = b^2 - 4ac$
$= 225 - 4(1)(45)$
$= 45$

$$x = \frac{-15 \pm \sqrt{45}}{2}$$

$\doteq -4.15$ or -10.85

length (x) cannot be negative

$\therefore x = 3$ is only solution.

$\therefore$ box is $3 \times 3 \times 15$

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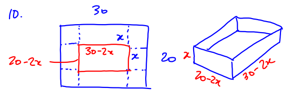
Assigned Work:

p.204 # 6, 7ad, 10, 13, 16

10.



$$V = x(30-2x)(20-2x)$$

$$\frac{1008}{2\cdot 2} = \frac{x(\cancel{2})(15-x)(\cancel{2})(10-x)}{\cancel{2}\cdot\cancel{2}}$$

$$252 = x(15-x)(10-x)$$

$$252 = x(150-25x+x^2)$$

$$252 = x^3-25x^2+150x$$

$$0 = x^3-25x^2+150x-252$$

let $V(x) = x^3-25x^2+150x-252$

$V(3)=0 \Rightarrow x-3$ is a factor

$$\begin{array}{r|rrrr} 3 & 1 & -25 & 150 & -252 \\ & \downarrow & 3 & -66 & 252 \\ \hline & 1 & -22 & 84 & 0 \end{array}$$

$$V(x) = (x-3)\underbrace{(x^2-22x+84)}_{\text{cannot factor}}$$

$x=3$

* set $V(x)=0$

QF?

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

⋮

$x \doteq 11+\sqrt{37}$ $\doteq 17.08$

$x = 11-\sqrt{37}$ $= 4.92$

but box exists in real world.

	$x=3$	$x=4.92$	$x=17$
$l = x$	3	4.92	17
$w = 30-2x$	24	20.16	~ -4
$h = 20-2x$	14	10.16	~ -14
	✓	✓	✗

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13. $d(t) = -3t^3+3t^2+18t$

$$= -3t(t^2-t-6)$$

$$= -3t(t-3)(t+2)$$

(b) set $d(t)=0$

$$0 = -3t(t-3)(t+2)$$

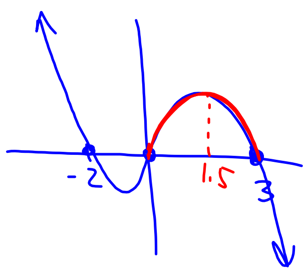
$t=0$ OR $t=3$ OR $t=-2$

Starts in harbour

returns to harbour

reject since $t \geq 0$

(d)



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16. $f(x) = a(x-2)(x-3)(x+5)$

Sub $P(4,36)$

$36 = a(4-2)(4-3)(4+5)$

$36 = a(2)(1)(9)$

$a = 2$

Determine where $(f(x))$ has a value of 120

Set $f(x) = 120$

$120 = 2(x-2)(x-3)(x+5)$

$60 = (x-2)(x-3)(x+5)$

$60 = (x-2)(x^2+2x-15)$

$60 = x^3-2x^2+2x^2-4x-15x+30$

$0 = x^3-19x-30$

let $g(x) = x^3-19x-30$

$\pm 1, 2, 3, 5, 6, 10, 15, 30$

$g(-2) = 0 \Rightarrow x+2$ is a factor

$$\begin{array}{r|rrrr} -2 & 1 & 0 & -19 & -30 \\ & \downarrow & -2 & 4 & 30 \\ \hline & 1 & -2 & -15 & 0 \end{array}$$

$g(x) = (x+2)(x^2-2x-15)$

$= (x+2)(x-5)(x+3)$

Set $g(x) = 0$

$0 = (x+2)(x-5)(x+3)$

$x = -2,\ x = 5,\ x = -3$

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