

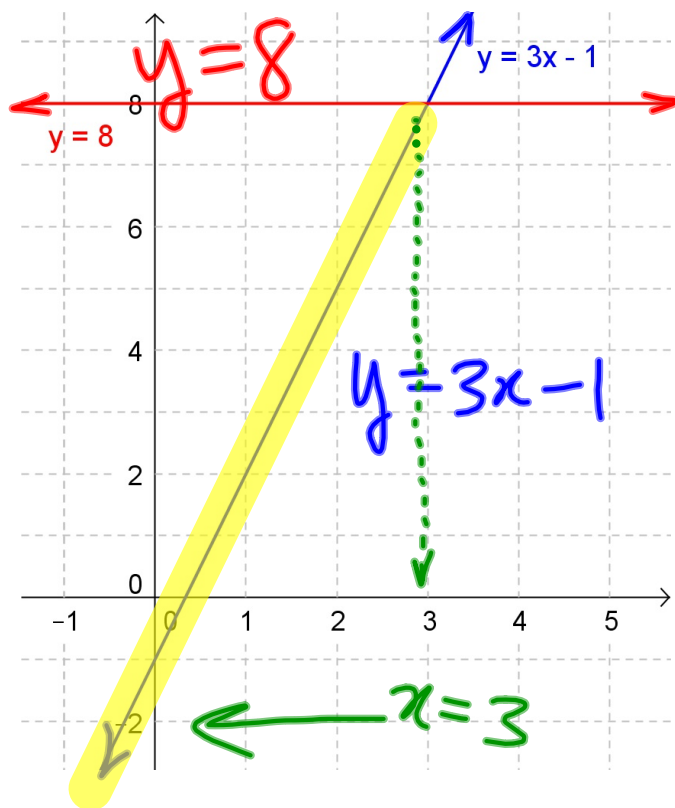
## Solving Linear Inequalities

Oct 6/2014

To solve an inequality, find all values that satisfy the inequality.

Consider:  $3x - 1 < 8$

The simplest way to visualize the solution is to graph and compare the LS and RS:



Where is the line  $y = 3x - 1$   
less than the line  $y = 8$ ?

all values where  
 $x < 3$

We have also solved such inequalities by:

- (1) solving the corresponding equation, then
- (2) testing values around the solution(s).

(1) Solve  $3x - 1 = 8$

$$3x = 9$$

$$x = 3$$

(2) Test  $x < \underline{3}$  and  $x > \underline{3}$

test  $x=0$

$$\begin{aligned} LS &= 3x - 1 & RS &= 8 \\ &= 3(0) - 1 \\ &= -1 \end{aligned}$$

$$LS < RS \checkmark$$

$\therefore x < 3$  is a  
solution

test  $x=4$

$$\begin{aligned} LS &= 3(4) - 1 \\ &= 12 - 1 \\ &= 11 \end{aligned}$$

$$RS = 8$$

$$\begin{aligned} LS &< RS? \\ 11 &< 8? \quad X \end{aligned}$$

$\therefore x > 3$  is not  
a solution

$$3x - 1 < 8$$

(1) Solve  $3x - 1 = 8$

$$3x = 9$$

$$x = 3$$

(2) Test  $x < 3$ :  $3(2) - 1 = 5$ , pass

Test  $x > 3$ :  $3(4) - 1 = 11$ , fail

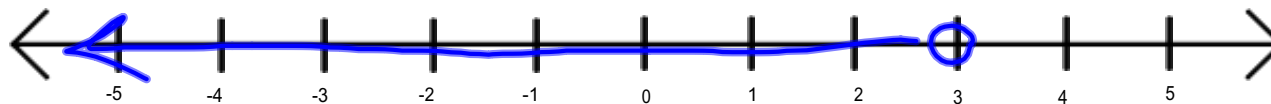
The solution can be represented as:

(a) set notation:  $\{x \in \mathbb{R} \mid x < 3\}$

(b) interval notation:  $x \in (-\infty, 3)$

(c) a number line:

$\in$  means "member of"



## Algebraic Operations on Inequalities

What are the effects of adding, subtracting, multiplying, and dividing on a very simple inequality?

Start with  $4 < 8$ , which is obviously true.

add positive:  $+5$

$$4 + 5 < 8 + 5 \\ 9 < 13 \checkmark$$

add negative:  $-3$

$$1 < 5 \checkmark$$

subtract positive:  $7$

$$-3 < 1 \checkmark$$

subtract negative:  $-4$

$$8 < 12 \checkmark$$

multiply by positive:  $3$

$$12 < 24 \checkmark$$

multiply by negative:  $-2$

$$-8 < -16 \times$$

divide by positive:  $4$

$$1 < 2 \checkmark$$

divide by negative:  $-1$

$$-4 < -8 \times$$

## Solving Inequalities Algebraically:

We can use the same basic operations (add, subtract, multiply, divide) that we would with a regular equation.

**Note: When multiplying or dividing by a negative value, the direction of the inequality must be switched.**

For a double-inequality, perform operations on all parts simultaneously.

Ex.1 Solve  $10 \leq 3(2x - 5) - (3x - 7) < 25$ .

Express your solution using:

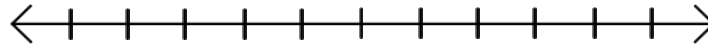
- (a) set notation,
- (b) interval notation,
- (c) a number line.



Ex.1 Solve  $10 \leq 3(2x - 5) - (3x - 7) < 25$ .

Express your solution using:

- (a) set notation,
- (b) interval notation,
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$$10 \leq 3(2x-5) - (3x-7) < 25$$

$$10 \leq 6x - 15 - 3x + 7 < 25$$

$$10 \leq 3x - 8 < 25$$

numbers      variables      numbers  
+8            +8            +8

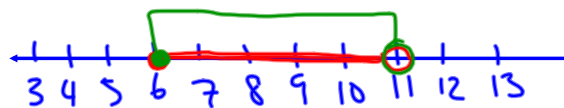
$$\frac{18}{3} \leq \frac{3x}{3} < \frac{33}{3}$$

$$6 \leq x < 11$$

(a)  $\{x \in \mathbb{R} \mid 6 \leq x < 11\}$

(b)  $x \in [6, 11)$

(c)



Assigned Work:

p.213 # 5bdf, 6be, 7bdf, 8, 9, 11, 15, 19

0 0 b c

$$5(f) \quad \frac{x-2}{3} \leq 2x-3 \quad [ \times 3 ]$$

$$x-2 \leq 6x-9 \quad \swarrow$$

$$\frac{-5x}{-5} \leq \frac{-7}{-5}$$
$$x \geq \frac{7}{5}$$

$$\frac{7}{5} \leq \frac{5x}{5}$$

$$\frac{7}{5} \leq x$$

$$x \geq \frac{7}{5} \quad \textcircled{c}$$

$$1 < x \leq 2 \quad \checkmark$$

$$2 \geq x > 1 \quad \textcircled{c}$$

$$6(e) \quad x(2x-1) \leq x+7$$

long way: solve, is 0 part  
of solution

Short way: test  $x=0$

$$\begin{array}{ll} \text{LS} = 0(2(0)-1) & \text{RS} = 0+7 \\ = 0 & = 7 \end{array}$$

$$0 \leq 7? \therefore$$

true, ✓

$\therefore 0$  is in the solution set.



8(b)

$$x \leq \frac{3}{2}$$

[ $\times 2$ ]

$$2x \leq 3$$

[ $\times (-1)$ ]

$$-2x \geq -3$$

[ $+7$ ]

$$-2x + 7 \geq 4$$

[ $-3x$ ]

$$-5x + 7 \geq -3x + 4$$

$$15. \quad |2x-1| < 7$$

$$-7 < 2x-1 < 7$$

+1    +1 +1

$$\frac{-6}{2} < \frac{2x}{2} < \frac{8}{2}$$

$$-3 < x < 4$$

$$|x| < 7$$

$$-7 < x < 7$$



$$|x| > 7$$

$$x > 7 \text{ or } x < -7$$



$$|2x-1| > 7$$

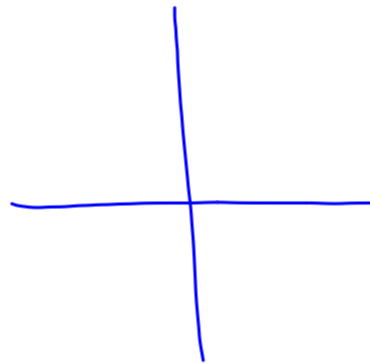
$$2x-1 > 7 \quad \text{or} \quad 2x-1 < -7$$

$$2x > 8$$

$$2x < -6$$

$$x > 4$$

$$x < -3$$



$$y = |2x-1|$$
$$= \left| 2\left(x - \frac{1}{2}\right) \right|$$

TDV.

$$19. (b) \quad 4x^2 + 5 \geq 41$$

$$4x^2 \geq 36$$

$$x^2 \geq 9$$

$$|x| \geq 3$$

$$x \geq 3 \text{ or } x \leq -3$$

$$x^2 = 9$$

$$|x| = 3$$

