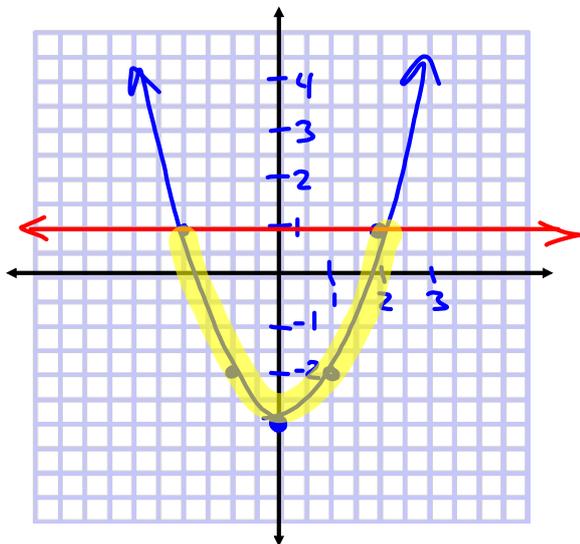


Solving Nonlinear Polynomial Inequalities

Oct 7/2014

(1) Graphically: Graph both sides of the inequality and compare.

Ex.1 Solve  $x^2 - 3 < 1$



where is  
 $y = x^2 - 3$   
 less than (below)  
 $y = 1$ ?  
 $-2 < x < 2$

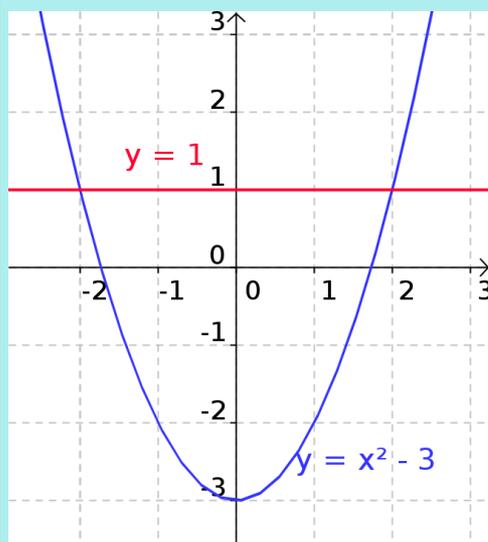
Oct 5-1:44 PM

Solving Nonlinear Polynomial Inequalities

Oct 7/2014

(1) Graphically: Graph both sides of the inequality and compare.

Ex.1 Solve  $x^2 - 3 < 1$



Where is the graph of

$y = x^2 - 3$   
 less than  $y = 1$

Oct 5-1:44 PM

(2) Algebraically:

- (a) Rearrange so one side is zero and solve the equation for both rational and irrational roots.  
 (b) Use the zeroes to create intervals.  
 (c) Test a value on each interval and see if it satisfies the original inequality.

Ex.2 Solve  $x^2 - 3 < 1$  algebraically

(a)  $x^2 - 4 < 0$  solve  $x^2 - 4 = 0$   
 $(x-2)(x+2) = 0$   
 $x = 2$  or  $x = -2$

(b)



(c) try  $x = -3$    try  $x = 0$    try  $x = 4$

$(-3)^2 - 4$	$0^2 - 4$	$4^2 - 4$
$= 9 - 4$	$= -4 < 0?$	$= 16 - 4$
$= 5 < 0?$	pass	$= 12 < 0?$
fails		fails

 $\therefore$  solution is  $-2 < x < 2$ 

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When examining intervals, we are only concerned with the sign, not the value, on each interval. It is often faster to use a factor table to test each interval.

Ex.3 Solve  $x^3 - 2x^2 + 5x + 20 \geq 2x^2 + 14x - 16$   
 (hint: factoring by grouping)

$$\underbrace{x^3 - 4x^2}_{x^2(x-4)} - \underbrace{9x + 36}_{-9(x+4)} \geq 0$$

$$\underbrace{x^2(x-4)}_{x^2(x-4)} - \underbrace{9(x+4)}_{-9(x+4)} \geq 0$$

↑ same! ↓

$$(x-4)(x^2-9) \geq 0$$

solve:  $(x-4)(x^2-9) = 0$

$$(x-4)(x-3)(x+3) = 0$$

$$x = 4 \text{ or } x = 3 \text{ or } x = -3$$

test:  $(x-4)(x-3)(x+3)$   
 for positives or negatives

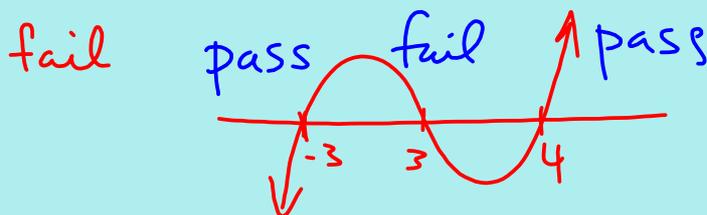
Oct 5-2:12 PM

$$x^3 - 2x^2 + 5x + 20 \geq 2x^2 + 14x - 16$$

$$(x - 4)(x - 3)(x + 3) \geq 0$$

-4 -3 0 3 3.5 4 5

factors	$x < -3$	$-3 < x < 3$	$3 < x < 4$	$x > 4$
$x - 4$	-	-	-	+
$x - 3$	-	-	+	+
$x + 3$	-	+	+	+
result $\rightarrow$	-	+	-	+



Solution is:  $-3 \leq x \leq 3$  or  $x \geq 4$

Oct 5-2:18 PM

Assigned Work:

p.225 # [1bd, 2, 3] fundamentals  
5, 6, 7, 10 (no graph or use tech) 8, 10, 11 15  
18 [not taking up]

desmos

6(d)  $(-3x)(x+7)(x-2) < 0$

zeros: 0, -7, 2

	-7	0	2	
	$x < -7$	$-7 < x < 0$	$0 < x < 2$	$x > 2$
$-3x$	+	+	-	-
$x+7$	-	+	+	+
$x-2$	-	-	-	+
result	+	-	+	-
	fail	pass	fail	pass

$\therefore$  solution is  $-7 < x < 0$  or  $x > 2$

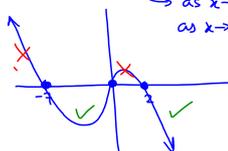
OR  
 $(-7, 0)$  OR  $(2, \infty)$

OR  
 $(-7, 0) \cup (2, \infty)$

$\uparrow$  "union"

Graph:  $y = -3x(x+7)(x-2)$  0:3

as  $x \rightarrow \infty, y \rightarrow -\infty$   
as  $x \rightarrow -\infty, y \rightarrow \infty$



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7/b)  $x^4 - 8x < 0$

solve:  $x^4 - 8x = 0$

$$x(x^3 - 8) = 0 \quad \begin{array}{l} A^3 - B^3 \\ = (A-B)(A^2 + AB + B^2) \end{array}$$

$A^3 - B^3$   
 $A = x \quad B = 2$

$$x(x-2)(x^2 + 2x + 4) = 0$$

cannot factor, but must test for irrational roots

$$b^2 - 4ac = 2^2 - 4(1)(4) = -12$$

Zeros: 0, 2

	$x < 0$	$0 < x < 2$	$x > 2$
$x$	-	+	+
$x-2$	-	-	+
$x^2 + 2x + 4$	+	+	+
	+	-	+

pass

Solution is:  $0 < x < 2$   
 $\{x \in \mathbb{R} \mid 0 < x < 2\}$   
 $x \in (0, 2)$

Oct 8-9:26 AM

10. order 4

$$f(x) > 0 \text{ for } -2 < x < 1$$

$$f(x) < 0 \text{ for } x < -2, x > 1$$

double root at  $x = 3$ 

$$f(-1) = 96$$

$$1 + 1 + 2 = 4$$

$$f(x) = a(x+2)^1(x-1)^1(x-3)^2$$

$$\text{sub } P(-1, 96)$$

$$\vdots$$

$$a = -3$$

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$$11. \quad v = -t^3 - 6t^2 + 12t + 50$$

$t$  is in  $50^\circ\text{C}$  increments

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