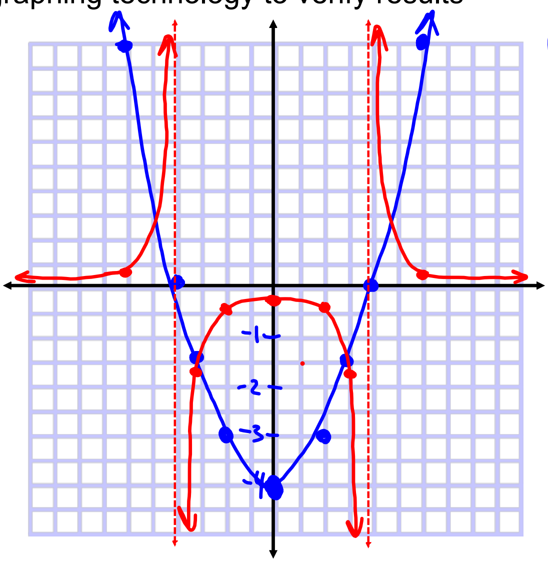
<u>Graphs of Reciprocal Functions</u>

The function $g(x)$ has a reciprocal function $f(x) = \dfrac{1}{g(x)}$

We shall limit $g(x)$ to polynomial functions for this unit.

(1) Do the "INVESTIGATE the Math" on p.248, parts F to H

- graph paper will be provided
- use graphing technology to verify results



$y = x^2 - 4$

$y = \dfrac{1}{x^2-4}$

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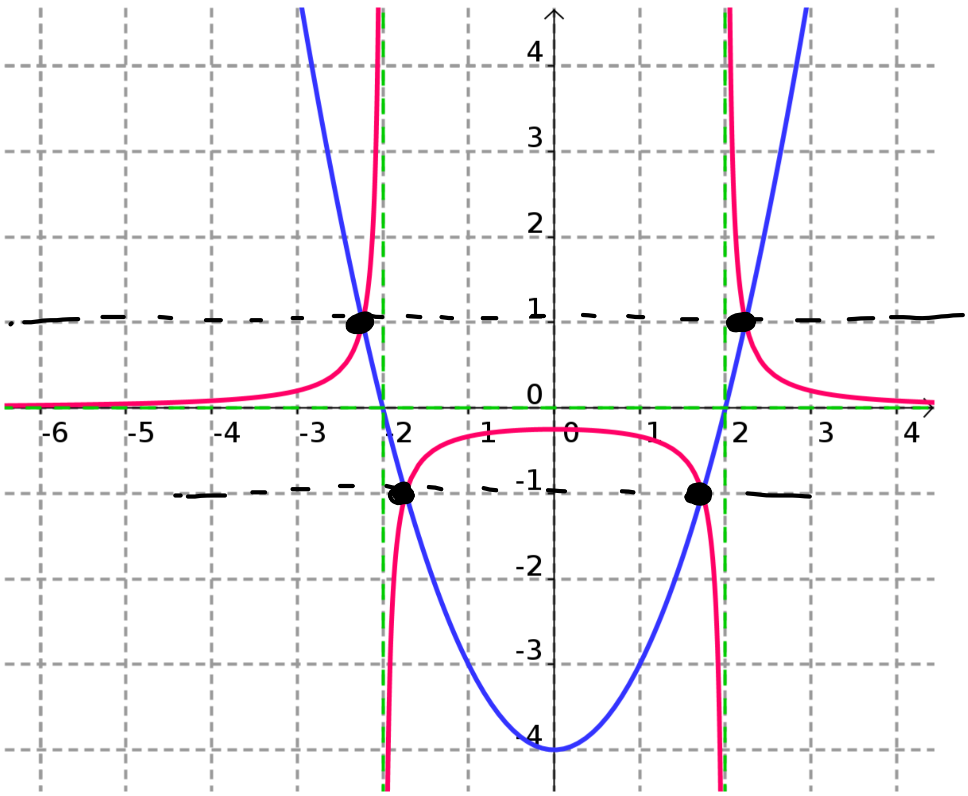

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characteristics	original	reciprocal
~~intercepts~~ & zeroes asymptotes	x-int : −2, 2	VA : $x=2$, $x=-2$
positive intervals	$(-\infty,-2)$ $(2,\infty)$	$(-\infty,-2)$ $(2,\infty)$
negative intervals	$(-2,2)$	$(-2,2)$
increasing intervals	$(0,\infty)$	$(-\infty,-2)$ $(-2,0)$
decreasing intervals	$(-\infty,0)$	$(0,2)$ $(2,\infty)$
points where y = 1	$x=\sqrt{5}$ $x=-\sqrt{5}$	$x=\sqrt{5}$ $x=-\sqrt{5}$
points where y = -1	$x=\pm\sqrt{3}$	$x=\pm\sqrt{3}$

exclude VA

Points of Intersection

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$\frac{\cancel{x}}{\cancel{x}} = 1$, $x \neq 0$, hole at $x=0$

$\frac{x}{x-1}$ VA : $x=1$

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Summary:

(a) if a point on a function has coordinates $\left(x, \frac{a}{b}\right)$

the reciprocal function has a point $\left(x, \frac{b}{a}\right)$

(b) if the original function has any zeroes, the reciprocal will have corresponding vertical asymptotes

(c) if the original function is linear or quadratic, its reciprocal will have a horizontal asymptote at y = 0

(d) the original and reciprocal will be positive and negative on the same intervals

(e) intervals of increase/decrease are reversed on reciprocal

(f) any local max/min points become local min/max points (they are reversed)

(g) any point on the original function with a y-value of 1 or -1 will intersect the reciprocal at that point

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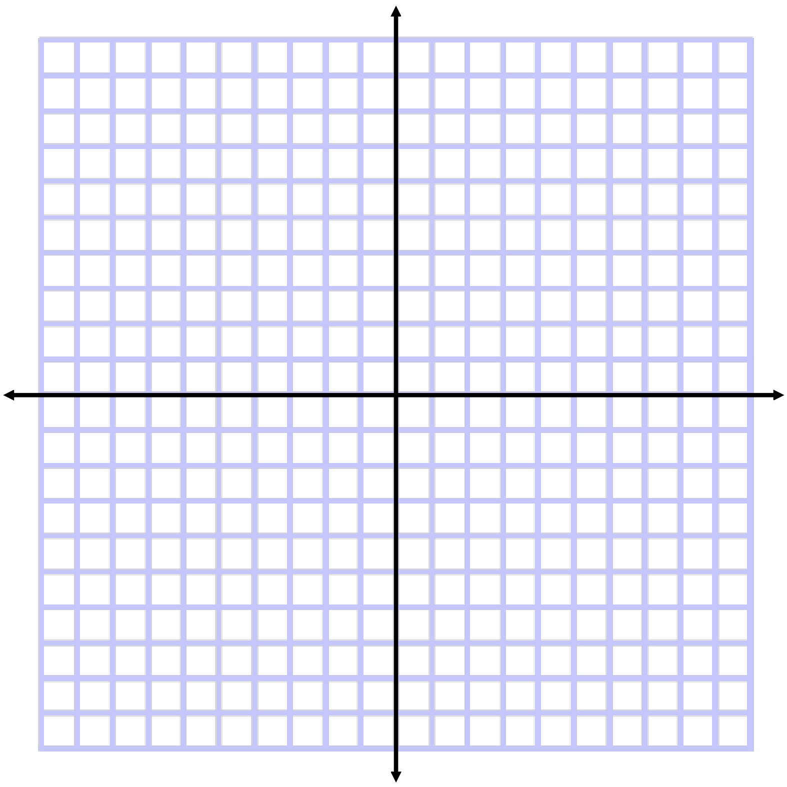
Ex.1 Graph $f(x) = (x+1)^2 - 4$ and $g(x) = \dfrac{1}{f(x)}$



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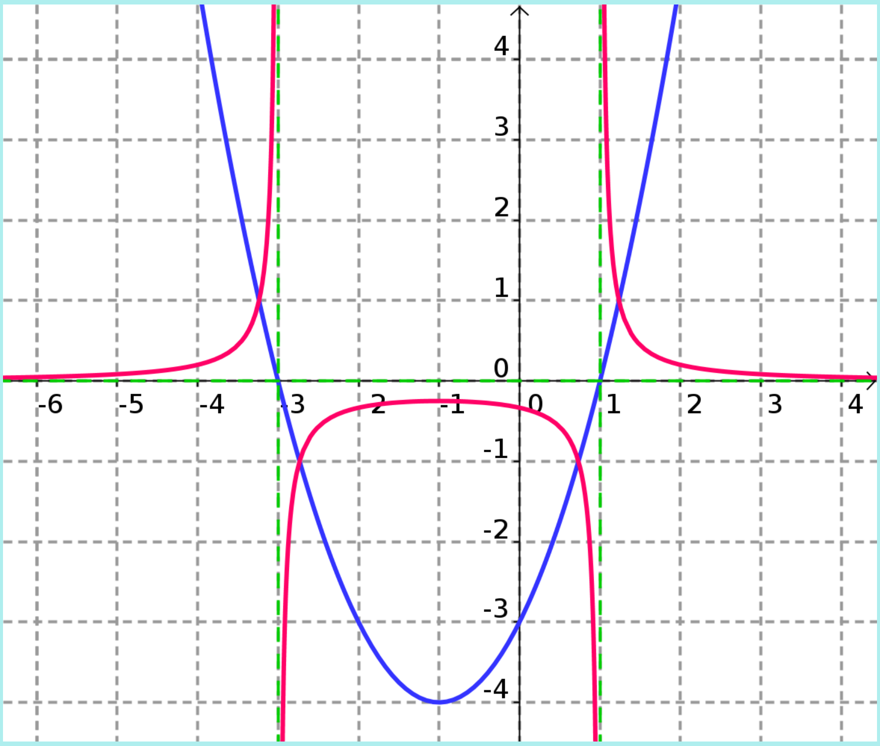
Ex.1 Graph $f(x) = (x+1)^2 - 4$ and $g(x) = \dfrac{1}{f(x)}$



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Assigned Work:

p.254 # 1, 2def, 6bcd, 8bdf, 9bc, 11

16 (find equation of reciprocal (shown) and original function)

2(e) $y = x^2 + 4$ $\quad y = \dfrac{1}{x^2+4}$

set $y = 0$

$0 = x^2 + 4$

$-4 = x^2$

$x = \pm\sqrt{-4}$ $\therefore$ no real roots

no VA

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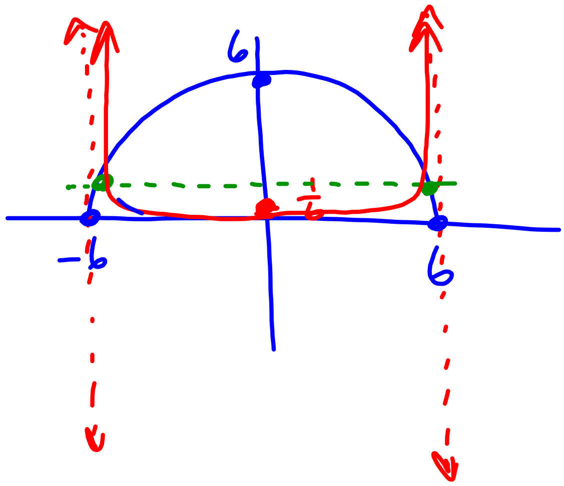
6(c)



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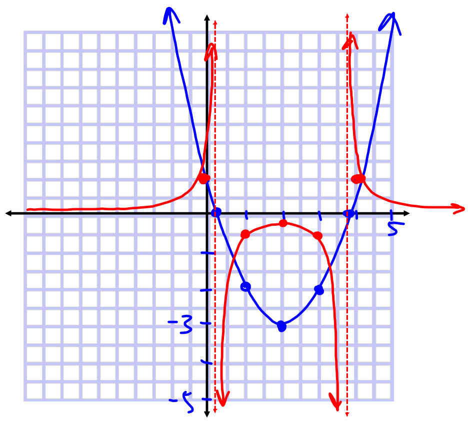
8(b) $f(x) = (x-2)^2 - 3$



$$(x-2)^2 - 3 = 0$$

$$(x-2)^2 = 3$$

$$x - 2 = \pm\sqrt{3}$$

$$\underbrace{x = 2 \pm \sqrt{3}}$$

VA: $x = 2+\sqrt{3}$, $x = 2-\sqrt{3}$

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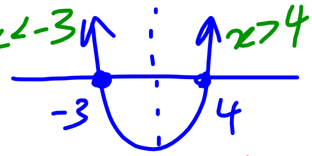
9(c)

$f(x) = x^2 - x - 12$
$= (x-4)(x+3)$

$D = \{x \in \mathbb{R}\}$



$-3 < x < 4$

A of S

zeroes: 4, -3

A of S: $\frac{4+(-3)}{2} = 0.5$

$y_{min} = f(0.5)$
$= -12.25$

$R = \{y \in \mathbb{R} \mid y \geq -12.25\}$

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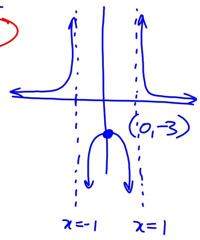
11. $y = \frac{k}{x^2+bx+c}$

zeroes → VA

$x=1,\ x=-1$

$\underbrace{\quad}$
$(x-1)(x+1)$
$= x^2 - 1$
$= x^2 + bx + c$

$b = 0$
$c = -1$



$y = \frac{k}{x^2-1}$

sub P(0,-3)

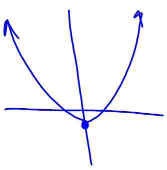
$-3 = \frac{k}{0^2-1}$

$-3 = \frac{k}{-1}$

$\boxed{k=3}$

$y = \frac{3}{x^2-1}$

$y = \frac{x^2-1}{3}$
$= \frac{1}{3}x^2 - \frac{1}{3}$



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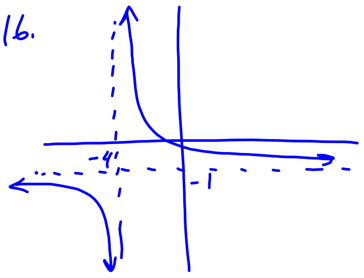
16.



VA: $x = -4 \longrightarrow x+4$ in denom.

$$y = \frac{k}{x+4} - 1$$

$$= \frac{1}{x+4} - \frac{1}{1}$$

$$= \frac{1}{x+4} - \frac{x+4}{x+4}$$

$$= \frac{-x-3}{x+4}$$

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