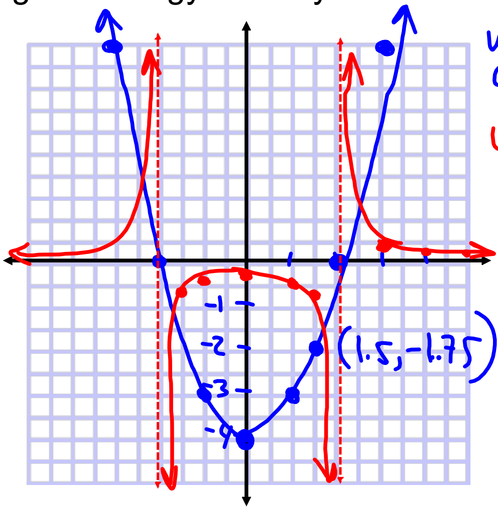
Graphs of Reciprocal Functions

The function $g(x)$ has a reciprocal function $f(x) = \dfrac{1}{g(x)}$

We shall limit $g(x)$ to polynomial functions for this unit.

(1) Do the "INVESTIGATE the Math" on p.248, parts F to H

- graph paper will be provided
- use graphing technology to verify results



$y = x^2 - 4$

$y = \dfrac{1}{x^2-4}$

$(0, -\frac{1}{4})$

$(1, -\frac{1}{3})$

$(-1, -\frac{1}{3})$

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$\dfrac{\quad}{0} \rightarrow$ undefined

hole : $\dfrac{0}{0}$

vertical asymptote : $\dfrac{k}{0}$

$k \neq 0$

e.g. $\dfrac{x}{x}$

$= \dfrac{1}{1}$

$= 1, \; x \neq 0$

$\dfrac{5}{x-1}$

VA: $x = 1$

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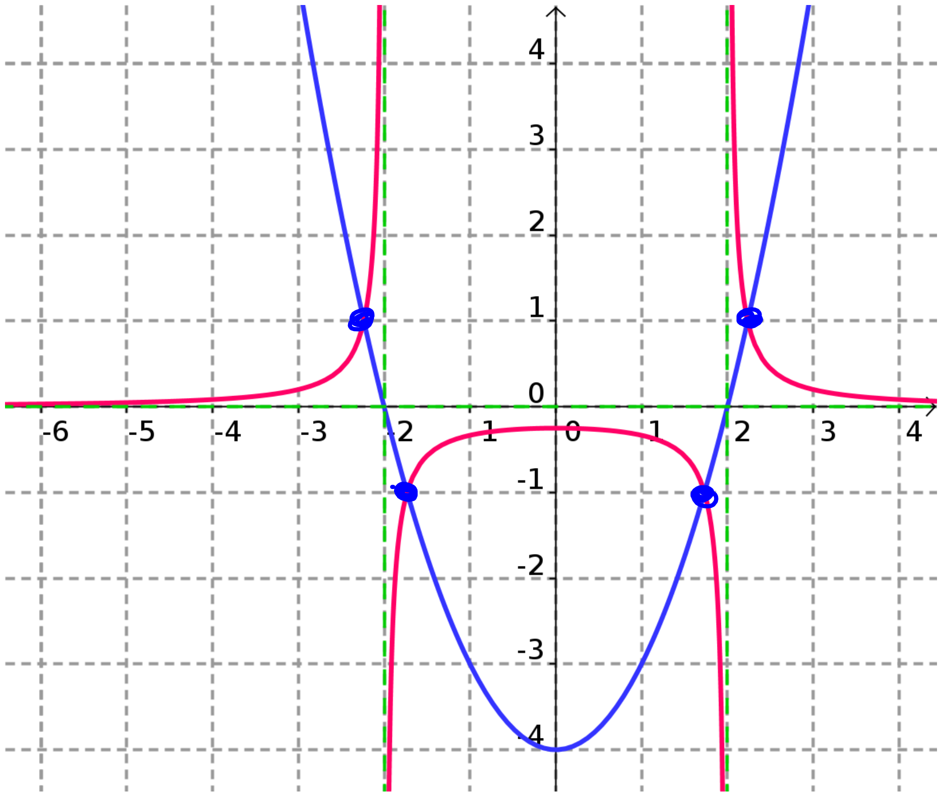

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characteristics	original	reciprocal
zeroes & asymptotes	$x = -2$ $x = 2$	VA: $x=-2$ $x=2$
positive intervals	$(-\infty,-2),(2,\infty)$	$(-\infty,-2),(2,\infty)$
negative intervals	$(-2, 2)$	$(-2, 2)$
increasing intervals	$(0, \infty)$	$(-\infty,-2)\cup(-2,0)$
decreasing intervals	$(-\infty, 0)$	$(0,2)\cup(2,\infty)$
points where y = 1		
points where y = -1		

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Summary:

(a) if a point on a function has coordinates $\left(x, \dfrac{a}{b}\right)$

the reciprocal function has a point $\left(x, \dfrac{b}{a}\right)$

(b) if the original function has any zeroes, the reciprocal will have corresponding vertical asymptotes

(c) if the original function is linear or quadratic, its reciprocal will have a horizontal asymptote at y = 0

(d) the original and reciprocal will be positive and negative on the same intervals

(e) intervals of increase/decrease are reversed on reciprocal

(f) any local max/min points become local min/max points (they are reversed)

(g) any point on the original function with a y-value of 1 or -1 will intersect the reciprocal at that point

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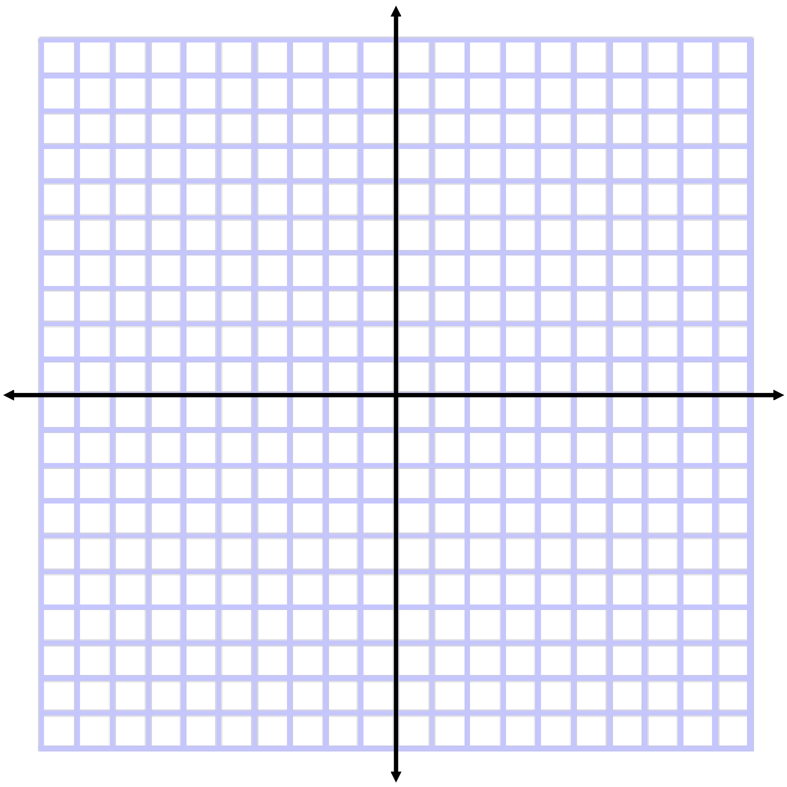
Ex.1 Graph $f(x) = (x+1)^2 - 4$ and $g(x) = \dfrac{1}{f(x)}$



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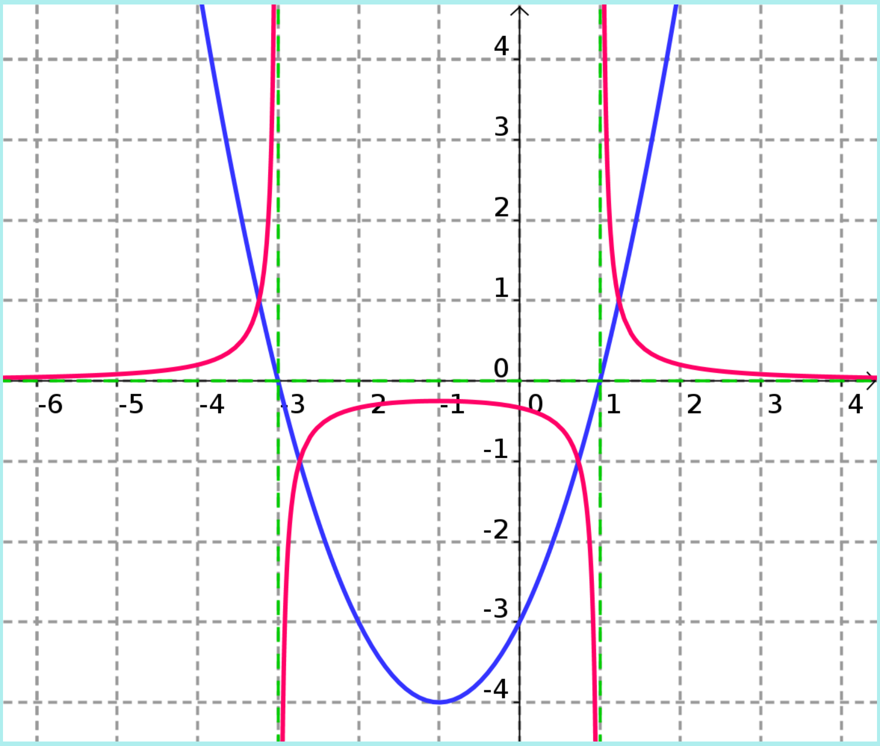
Ex.1 Graph $f(x) = (x+1)^2 - 4$ and $g(x) = \dfrac{1}{f(x)}$



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Assigned Work:

p.254 # 1, 2def, 6bcd, 8bdf, 9bc, 11
16 (find equation of reciprocal (shown) and original function)

2(d) $f(x) = 4x^2 - 25$ $\quad g(x) = \dfrac{1}{4x^2-25}$

$= (2x-5)(2x+5)$

zeroes: $x = \frac{5}{2}, -\frac{5}{2} \longrightarrow$ VA: $x = \frac{5}{2}$

$x = -\frac{5}{2}$

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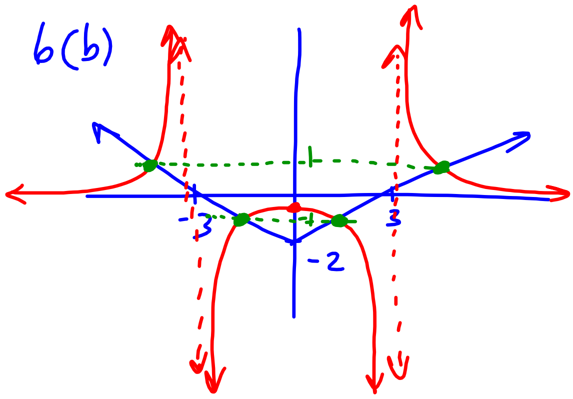
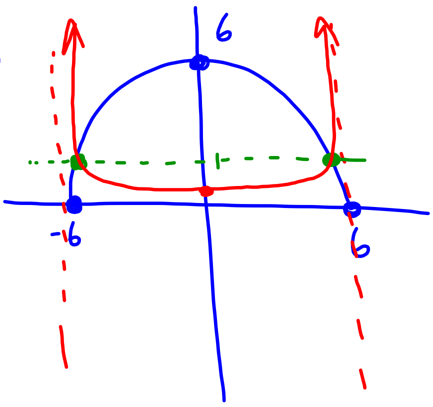
6(b)



(c)



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9(c) $f(x) = x^2 - x - 12$

$= (x-4)(x+3)$

$x = \frac{-b}{2a}$

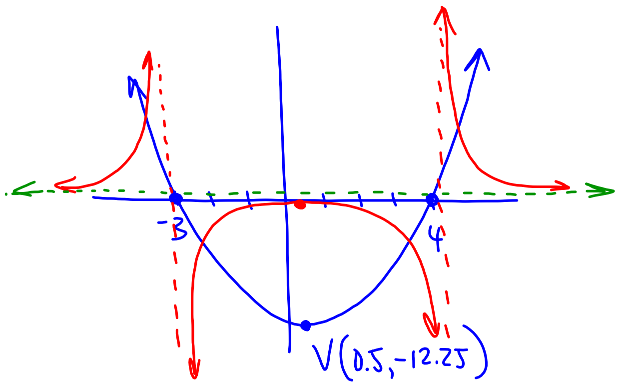
zeroes: 4, −3

A of S: $x = \frac{4+(-3)}{2}$

$x = 0.5$

Vertex $(0.5, f(0.5))$

$(0.5, -12.25)$



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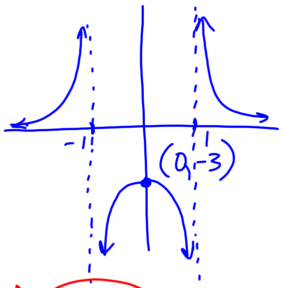
11. $y = \dfrac{k}{1x^2+bx+c}$



zeroes at 1, -1

VA: $x=1$
$x=-1$

$a(x+1)(x-1)$
$= a(x^2-1)$
$= x^2-1$

$y = \dfrac{k}{x^2-1}$

sub $(0,-3)$

$-3 = \dfrac{k}{0^2-1}$

$-3 = \dfrac{k}{-1}$

$\boxed{3 = k}$

$\therefore y = \dfrac{3}{x^2-1}$

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