Quotients of Polynomial Functions
(Rational Functions)

Oct 15/2014

Rational functions can be expressed as $f(x) = \dfrac{p(x)}{q(x)}$

where $p(x)$ and $q(x)$ are polynomial functions.

With the function q(x) in the denominator, we need to consider any discontinuities where q(x) = 0.

(a) 1 A hole will occur at $x = a$ if both p(x) and q(x) have a common factor of $(x - a)$.

(b) 2 A vertical asymptote will occur at $x = a$ when $\dfrac{p(a)}{q(a)} = \dfrac{k}{0}$ (i.e., a constant over zero).

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There are also horizontal and oblique asymptotes, which do not affect continuity. Instead, they determine the end behaviour of the rational function.

(3) The function $f(x) = \dfrac{p(x)}{q(x)}$ has a horizontal asymptote

if $\text{order of } p(x) \leq \text{order of } q(x)$

To determine the equation, divide the numerator by the denominator (long division, synthetic). Consider what happens as x gets very large (generally means we discard the remainder).

Ex.1 Determine the HA for $f(x) = \dfrac{2x}{x+1}$

$$\begin{array}{r} 2 \\ x+1 \,\big)\, \overline{2x} \\ \underline{2x+2} \\ -2 \end{array}$$

$$f(x) = \frac{2x}{x+1} = 2 - \frac{2}{x+1}$$

* end behaviour: what happens

as $x \to \infty$
as $x \to -\infty$ $\Big\}$ $f(x) \to 2$

HA: $y = 2$

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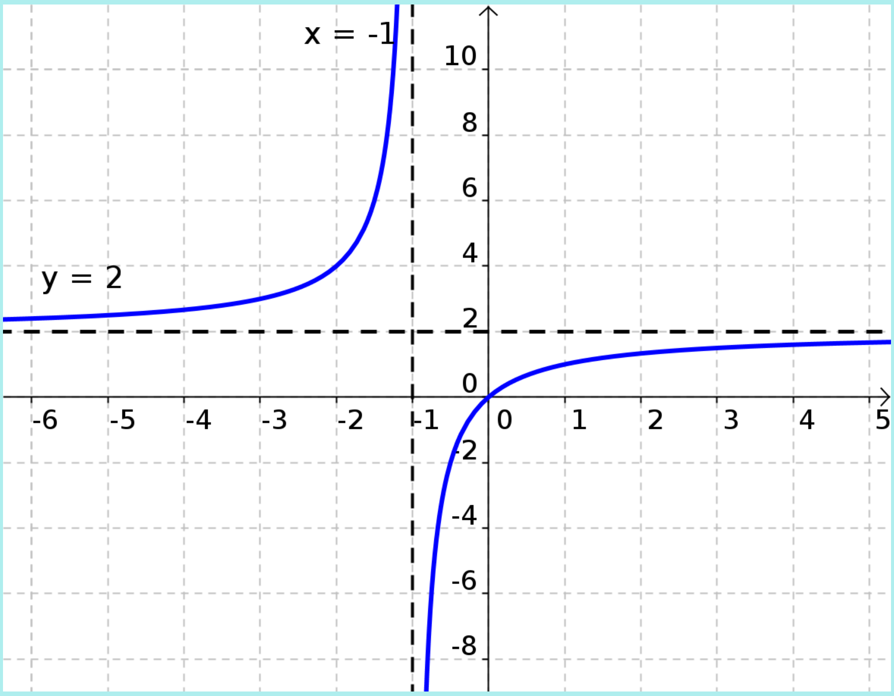
Ex.1 Determine the HA for $f(x) = \dfrac{2x}{x+1}$



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(4) An oblique asymptote will occur only if

$$\text{degree of } p(x) > \text{degree of } q(x) \text{ by exactly } 1$$

To determine the equation, divide the numerator by the denominator (long division, synthetic). Consider what happens as x gets very large (generally means we discard the remainder).

Ex.2 Determine the OA for $f(x) = \dfrac{x^2+4}{x+1}$

$$\begin{array}{r|l} & x - 1 \\ \hline x+1 & x^2 + 0x + 4 \\ & x^2 + x \\ \hline & \quad -x + 4 \\ & \quad -x - 1 \\ \hline & \qquad 5 \end{array}$$

$f(x) = x - 1 + \cancel{\dfrac{5}{x+1}} \to 0$

as $x \to \pm\infty$, remainder $\to 0$

OA : $y = x - 1$

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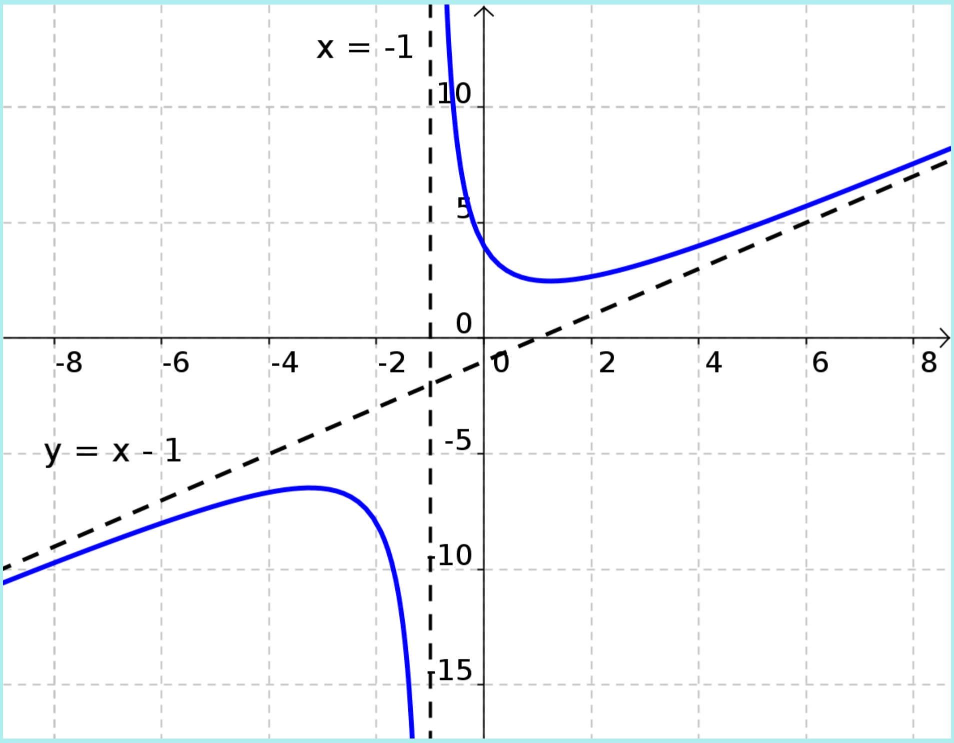
Ex.2 Determine the OA for $f(x) = \dfrac{x^2+4}{x+1}$



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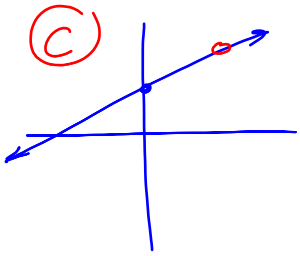
Assigned Work:

p.262 # 1, 2, 3

b g d
e g e
l

C



1(b) $y = \dfrac{x^2-9}{x-3}$

$= \dfrac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}}$

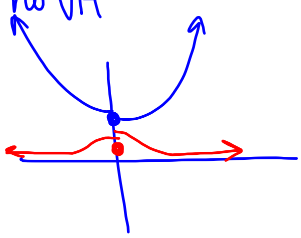
$= x+3$, hole at $x=3$

(e) $\dfrac{1}{x^2+5}$ x^2+5 S: 0 P = 5

no zeroes

no VA

B



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2 g)

2(g) $y = \frac{3x-6}{x-2}$

$= \frac{3\cancel{(x-2)}}{\cancel{(x-2)}}$

$\underbrace{y = 3}$ hole: $x = 2$

no HA, as we do not approach $y = 3$

Sub $x=2$, $y=3$

hole $\rightarrow (2,3)$

(f) $y = \frac{-3x+1}{2x-8}$

$= \frac{-3x+1}{2(x-4)}$

VA: $x = 4$

HA:
$$\begin{array}{r|l} & -\frac{3}{2} \\ 2x-8 & -3x+1 \\ & -3x+12 \\ \hline & -11 \rightarrow R \end{array}$$

$\frac{-3x+1}{2x-8} = -\frac{3}{2} + \frac{-11}{2x-8}$

HA: $y = -\frac{3}{2}$

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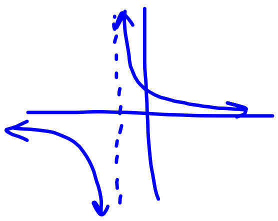
3(d) $\frac{1}{x+1}$ HA: $y = 0$



$\frac{1}{x+1} + 2$ HA: $y = 2$

$= \frac{1}{x+1} + \frac{2(x+1)}{x+1}$

$= \frac{2x+3}{x+1}$

$\frac{2}{3} + 1 = \frac{2}{3} + \frac{3}{3}$

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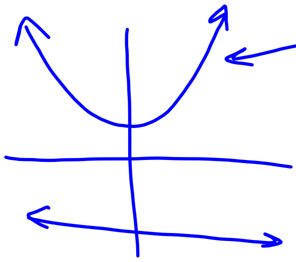
3(e) OA: $O_{top} - O_{bottom} = 1$

no VA : no zeroes in denominator



any even function $x^2 + 1$

$y = -3$

$$f(x) = \frac{x^3}{x^2+1}$$

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