Starting this Saturday, October 18th from 10AM to 12PM, and running once a month (sometimes twice), the Faculty of Engineering and Design at Carleton is running drop in tutorials to assist grade 12 students in Advanced Functions, Calculus and Physics.

http://carleton.ca/engineering-design/outreach/tutor/

Oct 15-11:48 AM

Graphs of Rational Functions

Oct 16/2014

Use these key characteristics to sketch a rational function:
(1) domain (in particular, discontinuities);
(2) x- and y-intercepts;
(3) asymptotes (vertical, horizontal, oblique), including behaviour near asymptotes; and
(4) positive and negative intervals.

Ex.1 Sketch the graph of $f(x) = \dfrac{4x-10}{2x+5}$

① $f(x) = \dfrac{2(2x-5)}{(2x+5)}$ $\therefore$ no holes

restriction: $2x+5=0$

$x = \frac{-5}{2}$

$D = \{x \in R \mid x \neq -\frac{5}{2}\}$

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Ex.1 Sketch the graph of $f(x) = \dfrac{4x-10}{2x+5}$

② y-intercept : set $x = 0$

avoid restrictions

$$f(0) = \frac{4(0)-10}{2(0)+5}$$

$$= -2$$

x-intercepts : set y (or $f(x)$) $= 0$

$$0 = \frac{4x-10}{2x+5}$$

$$(2x+5) \times 0 = \frac{2(2x-5)}{\cancel{2x+5}} \times \frac{\cancel{(2x+5)}}{1}$$

$$0 = 2(2x-5)$$

$$2x-5 = 0$$

$$x = \frac{5}{2} \Rightarrow \left(\frac{5}{2}, 0\right)$$

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Ex.1 Sketch the graph of $f(x) = \dfrac{4x-10}{2x+5}$

③ asymptotes

VA : zeroes of denominator (after removing holes)

$$2x+5 = 0$$

$$x = \frac{-5}{2}$$

HA/OA? $O_{num} = O_{den} \Rightarrow$ HA

HA : $y = 2$

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Ex.1 Sketch the graph of $f(x) = \dfrac{4x-10}{2x+5} = \dfrac{2(2x-5)}{2x+5}$

④ interval table

	$-\infty < x < -\frac{5}{2}$	$-\frac{5}{2}$ (VA)	$-\frac{5}{2} < x < \frac{5}{2}$	$\frac{5}{2}$ (x-int)	$x > \frac{5}{2}$
2	+		+		+
$2x-5$	−		−		+
$2x+5$	−		+		+
result	+		−		+

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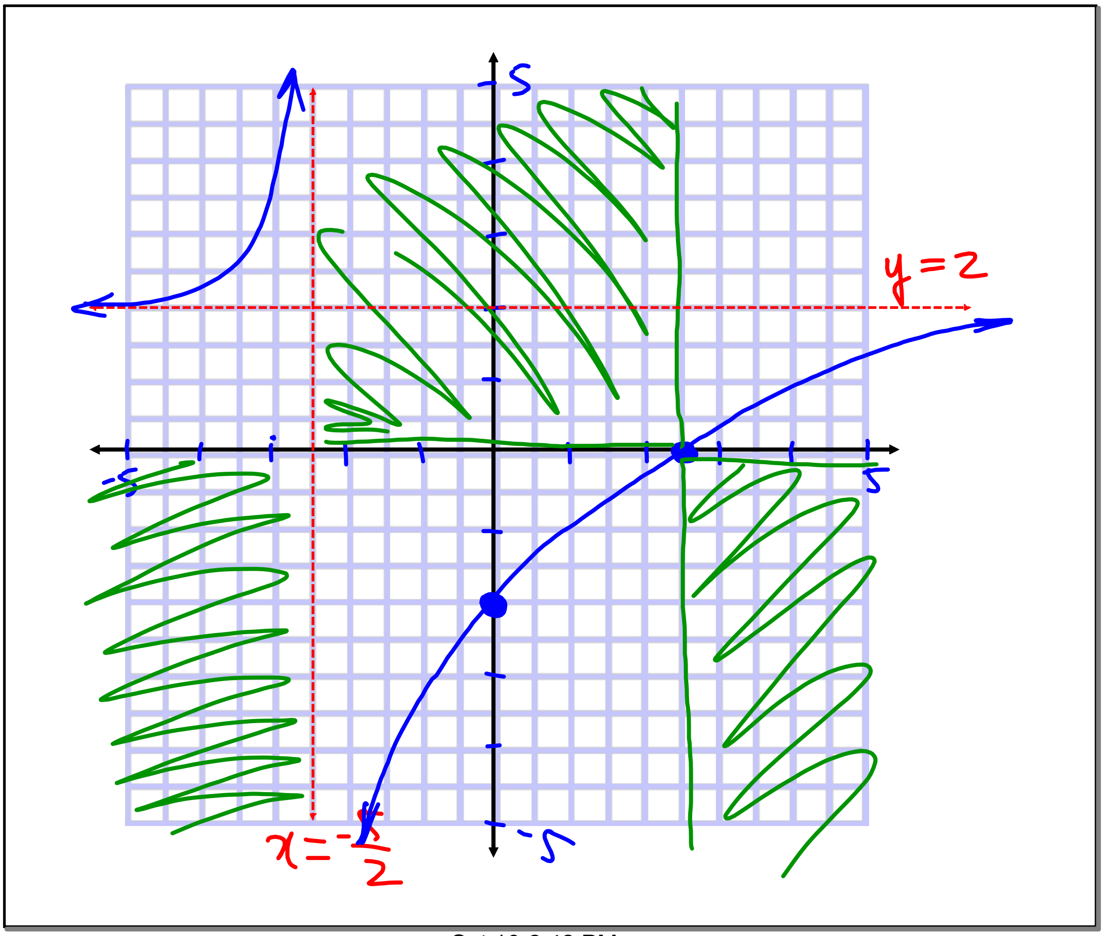

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Assigned Work:

p.272 # 1, 3, 5bcd, 6bd, 14

1. (b) $m(x) = \frac{2x-4}{x-2}$

$= \frac{2\cancel{(x-2)}}{\cancel{(x-2)}}$ C

$= 2, \quad x \neq 2$

(a) $h(x) = \frac{x+4}{2x+5} \rightarrow 2x+5=0$ A

$x = \frac{-5}{2}$ (VA)

(c) $f(x) = \frac{3}{x-1} \rightarrow$ VA: $x=1$ D

(d) $g(x) = \frac{2x-3}{x+2} \rightarrow$ VA: $x=-2$

B HA: $y=2$

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5(d) $f(x) = \frac{\cancel{x+2}^{\,1}}{5\cancel{(x+2)}_{\,1}}$ $\qquad \frac{\cancel{8}^{\,1}}{\cancel{8}_{\,1}}$

$f(x) = \frac{1}{5}, \quad \underbrace{x \neq -2}_{\text{hole at } (-2, \frac{1}{5})}$

$D = \{x \in \mathbb{R} \mid x \neq -2\}$

y-int: set $x=0$, $f(0) = \frac{1}{5}$

~~x-int~~: set $f(x)=0$. $0 = \frac{1}{5}$ not possible

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6(b) $f(x) = \frac{ax+b}{cx+d}$

VA : $x = -2$

Intuition
denom: $x+2$

$cx + d = 0$
$c(-2) + d = 0$
$-2c + d = 0$
$d = 2c$

HA : $y = 1$ ratio of leading coeffs

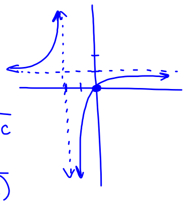
numerator
$x + b$

$\frac{a}{c} = 1$
$a = c$

y-int $= 0$, set $x = 0$, $f(0) = \frac{a(0)+b}{c(0)+d}$
$0 = \frac{b}{d}$
$b = 0$

x-int $= 0$, set $f(x) = 0$
$0 = \frac{ax+b}{cx+d}$
$0 = \frac{b}{d}$

$$\begin{aligned} f(x) &= \frac{ax+b}{cx+d} \\ &= \frac{ax}{cx+d} \\ &= \frac{cx}{cx+2c} \\ &= \frac{\cancel{c}x}{\cancel{c}(x+2)} \\ &= \frac{x}{x+2} \end{aligned}$$



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Many rational functions are in the form:
(linear divided by linear)

$$f(x) = \frac{ax+b}{cx+d}$$

Always start by looking for common factors and possible holes in graph.

Assuming no common factors between numerator and denominator, the equation of the horizontal asymptote will be:

$$\text{HA: } y = \frac{a}{c}$$

This can work with higher-order rational functions:

$$g(x) = \frac{3x^2 - 7x + 1}{2x^2 + 3x + 11} \qquad \text{HA: } y = \frac{3}{2}$$

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