

Transformations of Trigonometric Functions

Oct 30/2014

In general:  $y = af[k(x - p)] + q$

For trigonometric functions, this becomes

$$y = a \sin[k(x - p)] + q$$

or

$$y = a \cos[k(x - p)] + q$$

or

$$y = a \tan[k(x - p)] + q$$

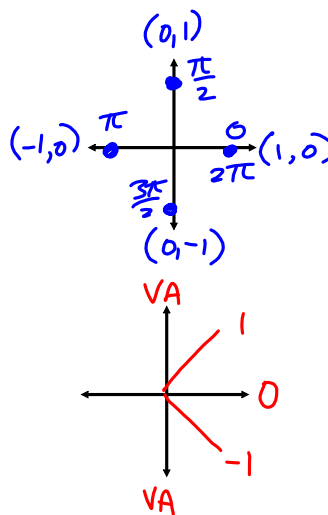
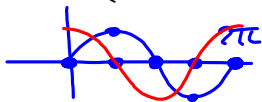
$$\begin{aligned} \sin(2x + \pi) \\ = \sin\left[2\left(x + \frac{\pi}{2}\right)\right] \end{aligned}$$

May 17-9:17 AM

(1) Graphing From Key Points

For sine and cosine, use points from the x- and y-axes on the unit circle.

$$\theta \in \left\{0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi\right\}$$

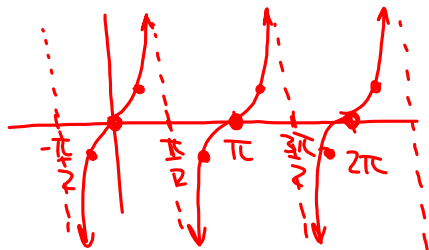


For tangent, use a cycle between two vertical asymptotes:

$$\theta \in \left\{\frac{-\pi}{2}, \frac{-\pi}{4}, 0, \frac{\pi}{4}, \frac{\pi}{2}\right\}$$

Transform each point:

$$(x, y) \rightarrow \left(\frac{x}{k} + p, ay + q\right)$$



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(2) Graphing From Key Properties

a - vertical reflection and amplitude

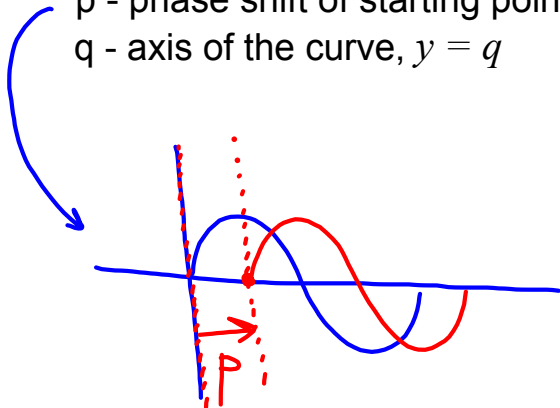
k - horizontal reflection and period

sine and cosine:  $\text{period} = \frac{2\pi}{k}$

tangent:  $\text{period} = \frac{\pi}{k}$

p - phase shift of starting point (from  $x = 0$ )

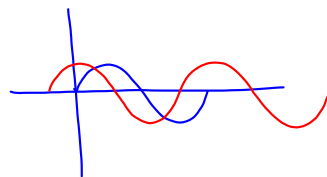
q - axis of the curve,  $y = q$



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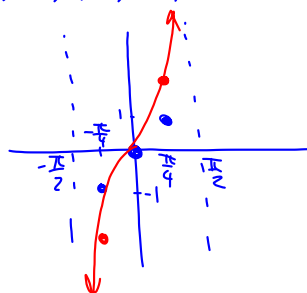
(3) Determining Equations of Transformed Functions

1. Select parent function (sine or cosine, tangent).
2. Identify key properties of period and axis of the curve and use them to determine k and q.
3. Identify key properties of amplitude and phase shift and use them to determine a and p.
4. Write the equation, remembering that multiple answers may correctly represent the same graph.



$n\pi$        $(n-1)\pi$

$\dots, -2\pi, -\pi, 0, \pi, 2\pi, \dots$        $\dots, -2\pi, -\pi, 0, \pi, 2\pi, \dots$



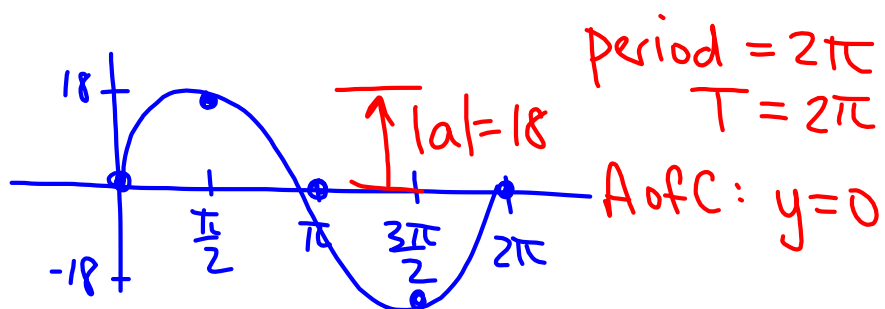
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Assigned Work:

p.343 # 1bd, 3, 4, 5 8ad, 9, 11, 12, 14

b

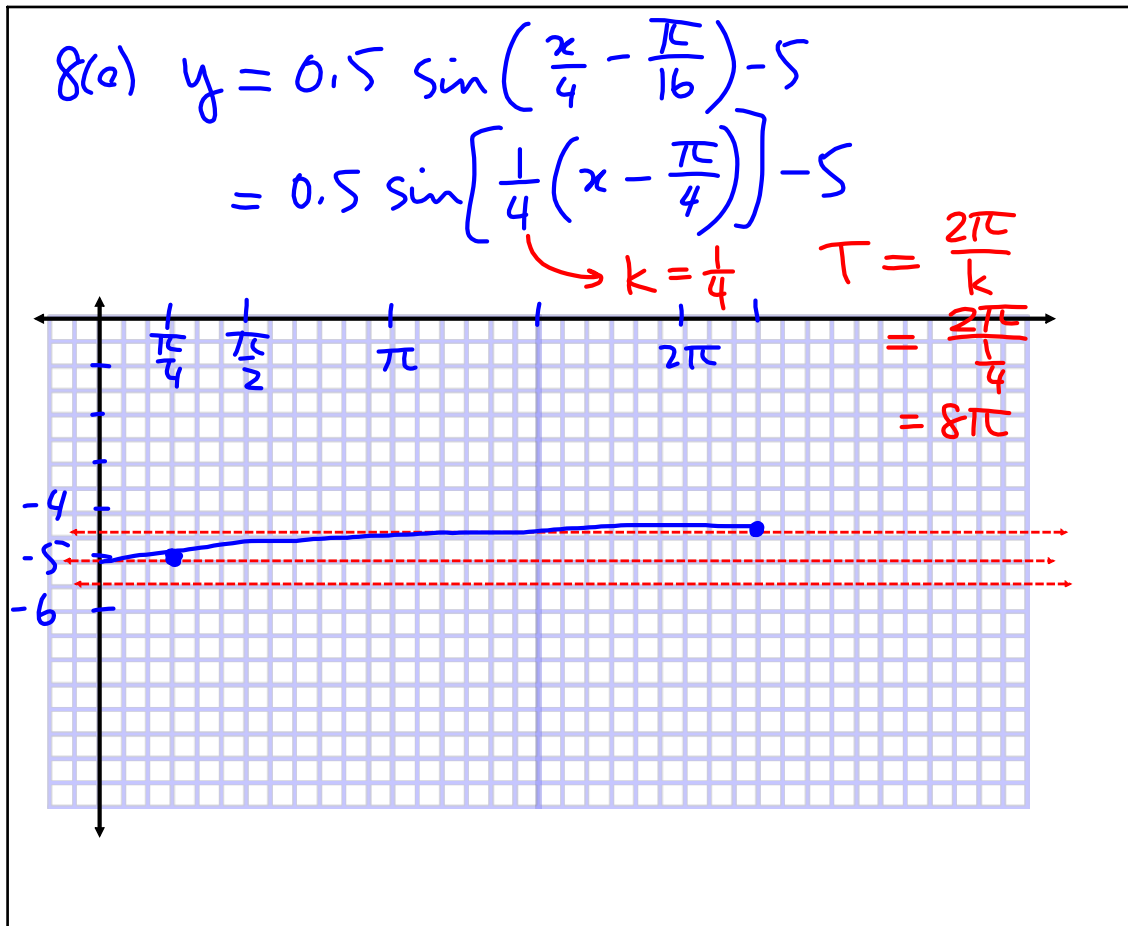
| $S(a)$ | $x$ | $0$  | $\frac{\pi}{2}$ | $\pi$ | $\frac{3\pi}{2}$ | $2\pi$ |
|--------|-----|------|-----------------|-------|------------------|--------|
| $y$    | $0$ | $18$ | $0$             | $-18$ | $0$              |        |



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8(e)

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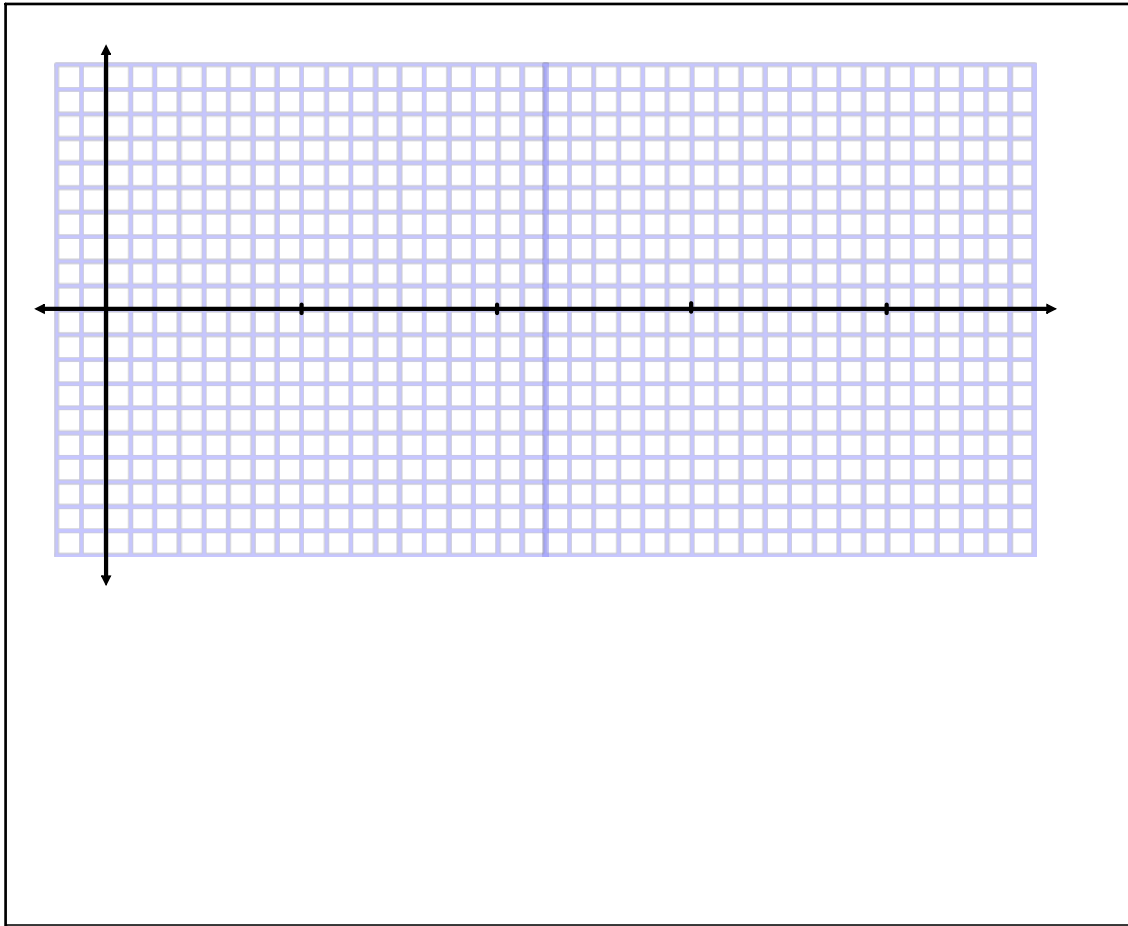
9.  $P(t) = -20 \cos\left(\frac{5\pi}{3}t\right) + 100$

period =  $\frac{2\pi}{k}$   
 $= \frac{2\pi}{\frac{5\pi}{3}}$   
 $= 2\pi \times \frac{3}{5\pi}$   
 $= \frac{6}{5}$  (seconds)

$\frac{6}{5}$  seconds / beat      60 seconds / minute

# of beats / minute =  $\left(\frac{5 \text{ beats}}{6 \text{ seconds}}\right) \left(\frac{60 \text{ seconds}}{1 \text{ minute}}\right)$   
 $= 50$  bpm

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