

Unit 5: Trigonometric Identities & Equations

Equivalent Trigonometric Functions

Nov. 7/2014

Due to the periodic nature of trigonometric functions, there are multiple (infinite) ways to express equivalent functions.

(1) Using the period:

Both sine and cosine have a period of 2π , which means any phase shift by a multiple of the period will be equivalent.

$$\sin(\theta) = \sin(\theta + 2\pi) = \sin(\theta - 2\pi)$$

$$\cos(\theta) = \cos(\theta + 2\pi) = \cos(\theta - 2\pi)$$

Similarly, for tangent,

$$\tan(\theta) = \tan(\theta + \pi) = \tan(\theta - \pi)$$

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(2) By symmetry:

Recall, even functions: $f(x) = f(-x)$

odd functions: $f(-x) = -f(x)$

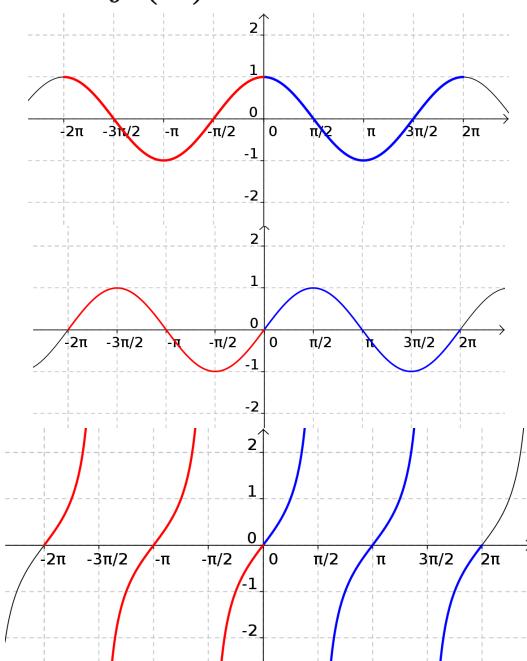
Cosine is even (reflective symmetry across the y-axis)

$$\cos(x) = \cos(-x)$$

Sine and tangent are odd (rotational symmetry)

$$\sin(-x) = -\sin(x)$$

$$\tan(-x) = -\tan(x)$$



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(3) Using complimentary angles:

Recall: Complimentary angles add to $\frac{\pi}{2}$ (or 90°)

$$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2} = \cos\left(\frac{\pi}{6}\right)$$

$$\cos\left(\frac{\pi}{3}\right) = \frac{1}{2} = \sin\left(\frac{\pi}{6}\right)$$

$$\tan\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{1} = \cot\left(\frac{\pi}{6}\right)$$

$$\sec\left(\frac{\pi}{3}\right) = \frac{2}{1} = \csc\left(\frac{\pi}{6}\right)$$

In general:

$$\sin\theta = \cos\left(\frac{\pi}{2} - \theta\right)$$

$$\tan\theta = \cot\left(\frac{\pi}{2} - \theta\right)$$

$$\csc\theta = \sec\left(\frac{\pi}{2} - \theta\right)$$

$0 < \theta < \frac{\pi}{2}$ (θ must fit inside a right triangle)

$$\cos\left(\frac{\pi}{2} - \theta\right) = \cos(-\theta + \frac{\pi}{2})$$

$$\sin\theta = \cos\left[-1\left(\theta - \frac{\pi}{2}\right)\right]$$

$\theta \in \mathbb{R}$

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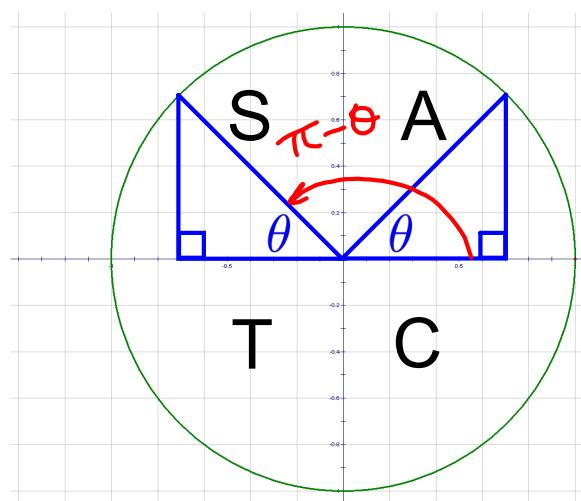
(4) Using primary and related acute angles:

The trigonometric ratio for an angle in any quadrant can be expressed using the RAA and the CAST rule.

$$\sin(\pi - \theta) = + \sin \theta$$

$$\cos(\pi - \theta) = - \cos \theta$$

$$\tan(\pi - \theta) = - \tan \theta$$



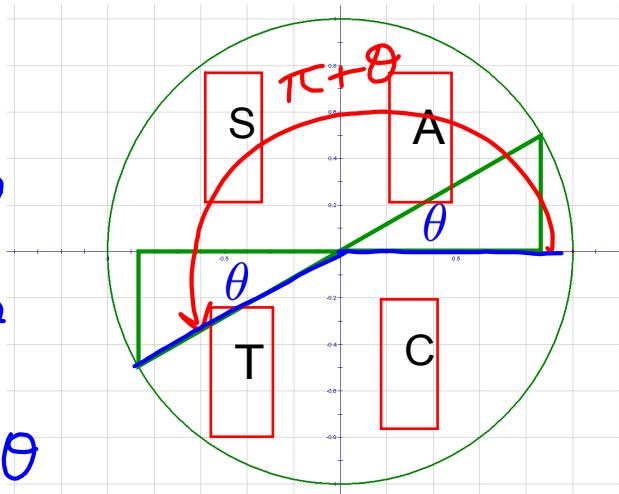
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In Q3:

$$\sin(\pi + \theta) = -\sin \theta$$

$$\cos(\pi + \theta) = -\cos \theta$$

$$\tan(\pi + \theta) = +\tan \theta$$

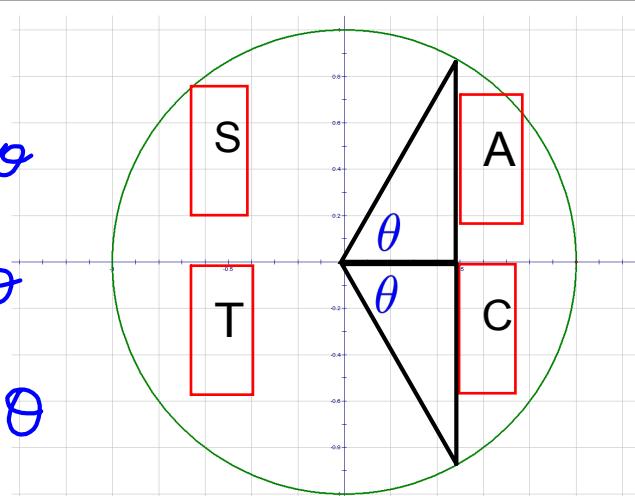


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$$\sin(2\pi - \theta) = -\sin \theta$$

$$\cos(2\pi - \theta) = +\cos \theta$$

$$\tan(2\pi - \theta) = -\tan \theta$$

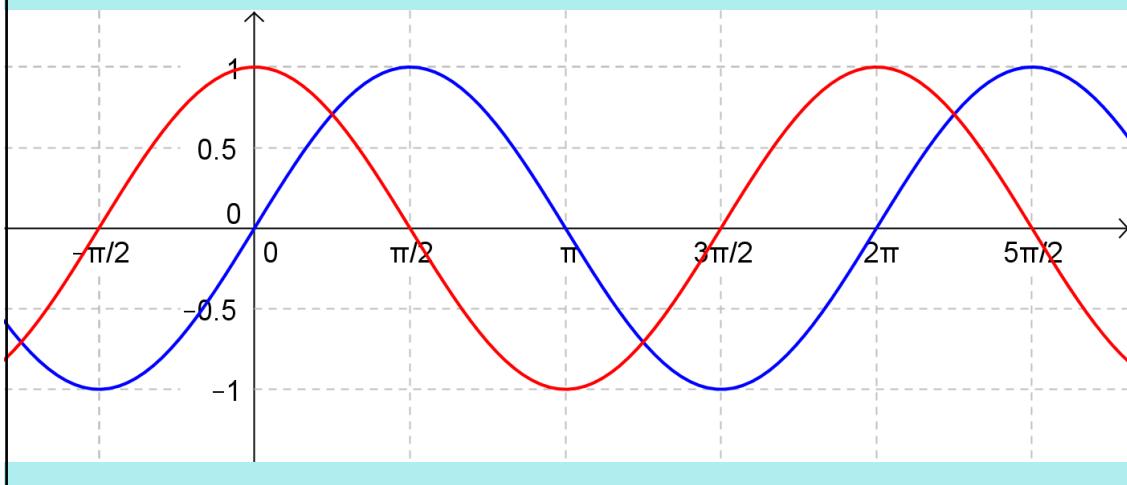


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(5) By transformations (reflections & phase shift):

Due to the periodic nature of all graphs, and how certain pairs are so similar (sine/cosine, tangent/cotangent, secant/cosecant), it is possible to verify equivalent expressions from the graphs through an application of transformations.

Ex.1 Verify



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Assigned Work:

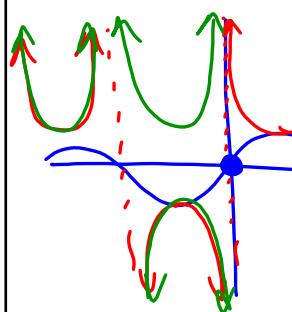
p.392 # 1, 2a, 3acd, 4a, 5abc

$$2(a) \text{ even : } f(x) = f(-x)$$

$$\text{odd : } f(x) = -f(-x)$$

or

$$f(-x) = -f(x)$$



$\csc \theta$ is odd

$$\cos \theta = -\csc(-\theta)$$

or

$$\csc(-\theta) = -\csc \theta$$

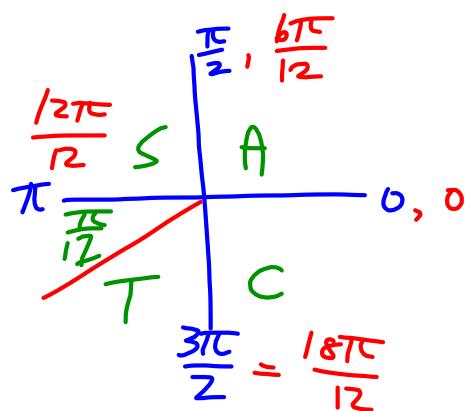
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$$\begin{aligned}
 3(c) \quad \tan\left(\frac{3\pi}{8}\right) &= \cot\left(\frac{\pi}{2} - \frac{3\pi}{8}\right) \\
 &= \cot\left(\frac{4\pi}{8} - \frac{3\pi}{8}\right) \\
 &= \cot\left(\frac{\pi}{8}\right)
 \end{aligned}$$

$$\begin{aligned}
 3(d) \quad \cos\left(\frac{5\pi}{16}\right) &= \sin\left(\frac{\pi}{2} - \frac{5\pi}{16}\right) \\
 &= \sin\left(\frac{8\pi}{16} - \frac{5\pi}{16}\right) \\
 &= \sin\left(\frac{3\pi}{16}\right)
 \end{aligned}$$

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$$5(b) \quad \cos\left(\frac{13\pi}{12}\right) = -\cos\left(\frac{\pi}{12}\right)$$



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