Unit 5: Trigonometric Identities & Equations

Equivalent Trigonometric Functions Nov.7/2014

Due to the periodic nature of trigonometric functions, there are multiple (infinite) ways to express equivalent functions.

(1) Using the period:

Both sine and cosine have a period of 2π , which means any phase shift by a multiple of the period will be equivalent.

$$\sin(\theta) = \sin(\theta + 2\pi) = \sin(\theta - 2\pi)$$
$$\cos(\theta) = \cos(\theta + 2\pi) = \cos(\theta - 2\pi)$$

Similarly, for tangent,

$$\tan(\theta) = \tan(\theta + \pi) = \tan(\theta - \pi)$$

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(2) By symmetry:

Recall, even functions: $f(x) = f(-x)$

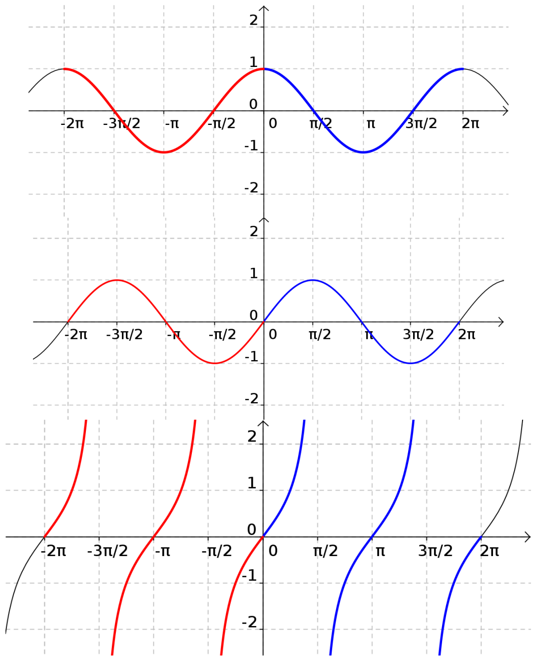
odd functions: $f(-x) = -f(x)$

Cosine is even (reflective symmetry across the y-axis)

$$\cos(x) = \cos(-x)$$

Sine and tangent are odd (rotational symmetry)

$$\sin(-x) = -\sin(x)$$
$$\tan(-x) = -\tan(x)$$



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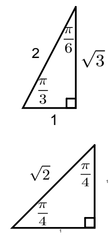
(3) Using complimentary angles:

Recall: Complimentary angles add to $\frac{\pi}{2}$ (or 90°)

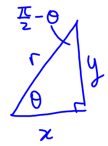


$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2} = \cos\left(\frac{\pi}{6}\right)$

$\cos\left(\frac{\pi}{3}\right) = \frac{1}{2} = \sin\left(\frac{\pi}{6}\right)$

$\tan\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{1} = \cot\left(\frac{\pi}{6}\right)$

In general,



$\sin\theta = \cos\left(\frac{\pi}{2}-\theta\right)$

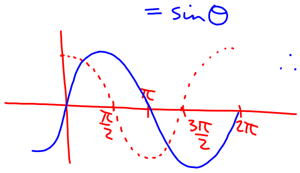
$\cos\theta = \sin\left(\frac{\pi}{2}-\theta\right)$

$\tan\theta = \cot\left(\frac{\pi}{2}-\theta\right)$

$\csc\theta = \sec\left(\frac{\pi}{2}-\theta\right)$

Shown for $0<\theta<\frac{\pi}{2}$ (inside right triangle)

consider: $y = \cos\left(\frac{\pi}{2}-\theta\right)$
$= \cos\left(-\theta+\frac{\pi}{2}\right)$
$= \cos\left[-1\left(\theta-\frac{\pi}{2}\right)\right]$
$= \sin\theta$



$\therefore \theta \in \mathbb{R}$

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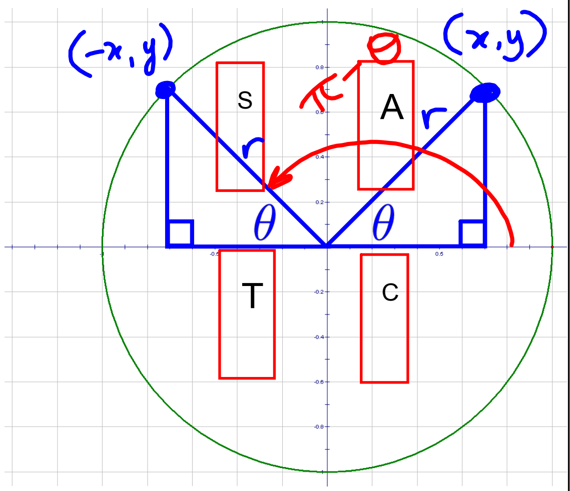
(4) Using primary and related acute angles:

The trigonometric ratio for an angle in any quadrant can be expressed using the RAA and the CAST rule.

$\sin(\pi-\theta) = +\sin\theta$

$\cos(\pi-\theta) = -\cos\theta$

$\tan(\pi-\theta) = -\tan\theta$



$\sin\theta = \frac{y}{r}$

$\sin(\pi-\theta) = \frac{y}{r}$

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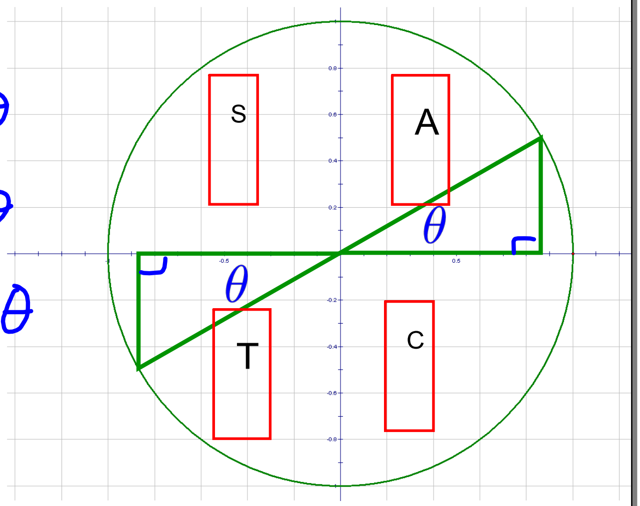
in Q3:

$\sin(\pi+\theta) = -\sin\theta$

$\cos(\pi+\theta) = -\cos\theta$

$\tan(\pi+\theta) = +\tan\theta$



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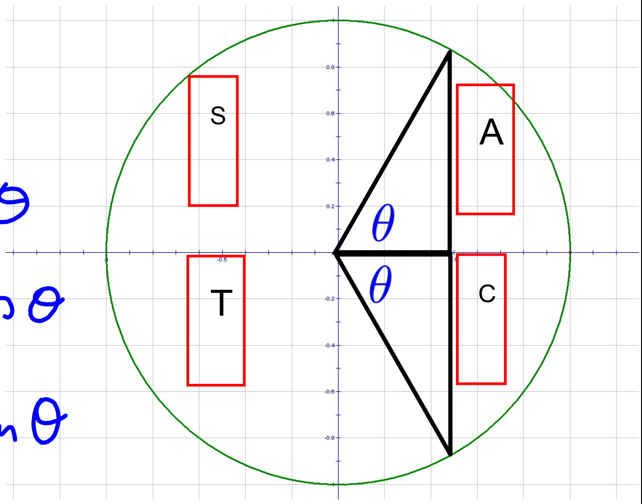
in Q4:

$\sin(2\pi-\theta) = -\sin\theta$

$\cos(2\pi-\theta) = +\cos\theta$

$\tan(2\pi-\theta) = -\tan\theta$



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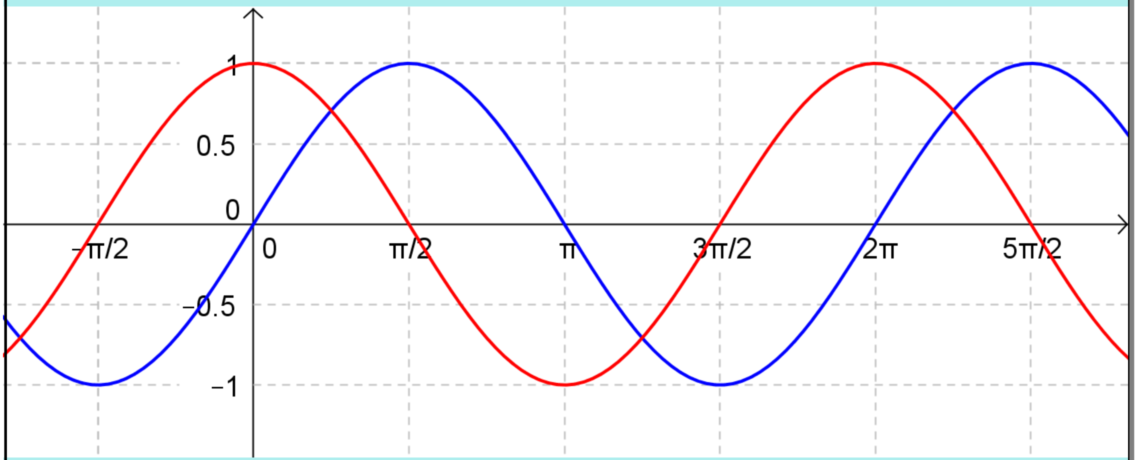
(5) By transformations (reflections & phase shift):

Due to the periodic nature of all graphs, and how certain pairs are so similar (sine/cosine, tangent/cotangent, secant/cosecant), it is possible to verify equivalent expressions from the graphs through an application of transformations.

Ex.1 Verify



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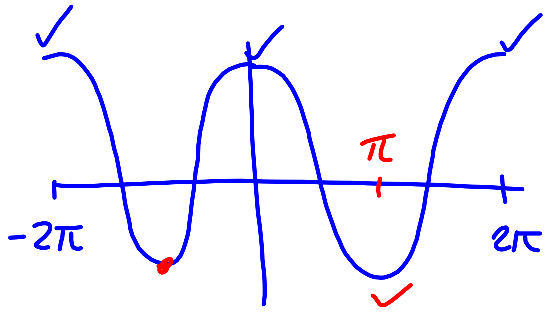
Assigned Work:

p.392 # 1, 2a, 3acd, 4a, 5abc

a

1(a)



$y = \cos\theta$

$y = \cos(\theta - 2\pi)$

$y = \cos(\theta + 2\pi)$

$y = -\cos(\theta - \pi)$

$y = -\cos(\theta + \pi)$

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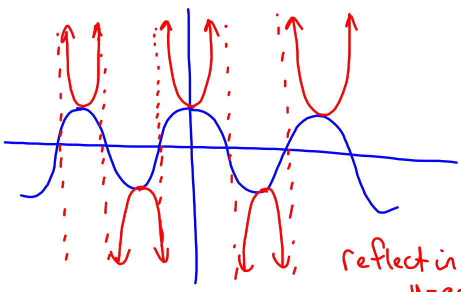
2(a) $y = \sec\theta$



reflect in y-axis

$\Rightarrow$ even function

even: $f(x) = f(-x)$

$\sec(x) = \sec(-x)$

odd: $f(-x) = -f(x)$

or

$f(x) = -f(-x)$

$\csc(x) = -\csc(-x)$

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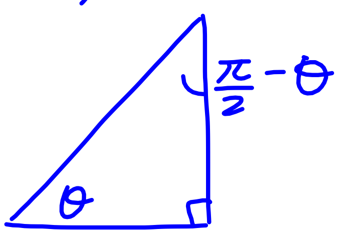
3(c) $\tan\left(\frac{3\pi}{8}\right) = \cot\left(\frac{\pi}{2} - \frac{3\pi}{8}\right)$

$= \cot\left(\frac{4\pi}{8} - \frac{3\pi}{8}\right)$

$= \cot\left(\frac{\pi}{8}\right)$



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4 (a) $\csc(\theta) = \sec\left(\frac{\pi}{2} - \theta\right)$

$$\frac{1}{\sin\theta} = \frac{1}{\cos\left(\frac{\pi}{2} - \theta\right)}$$

$$\sec(\theta) = \csc\left(\frac{\pi}{2} - \theta\right)$$

$$\cot(\theta) = \tan\left(\frac{\pi}{2} - \theta\right)$$

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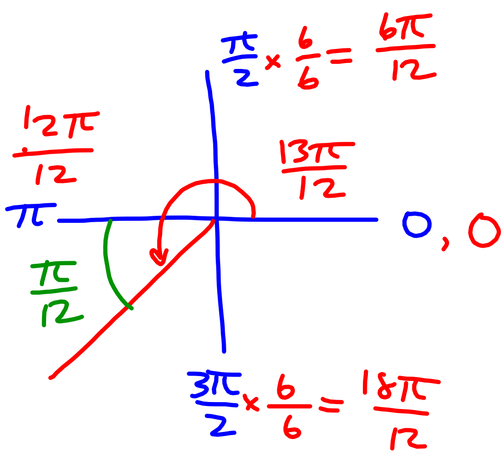
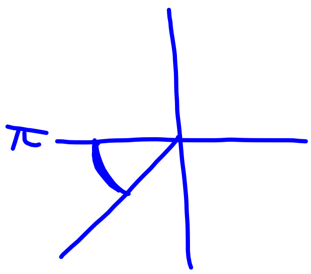
5 (b) $\cos\left(\frac{13\pi}{12}\right) = -\cos\left(\frac{\pi}{12}\right)$





(c) $\tan\left(\frac{5\pi}{4}\right) = +\tan\left(\frac{\pi}{4}\right)$

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