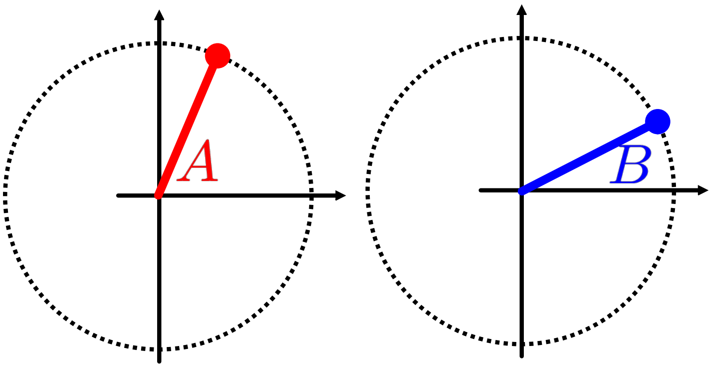
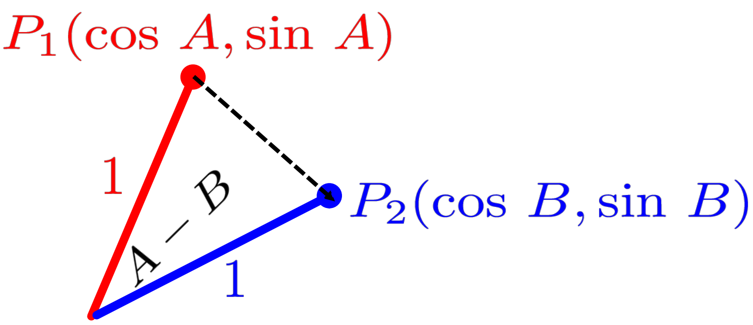
Compound Angle Formulas

Nov. 13/2014

Consider the angles 'A' and 'B' on the unit circle.





Nov 6-7:25 PM

Cosine Law: $c^2 = a^2 + b^2 - 2ab\cos C$

Distance Formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Use cosine law and distance formula to develop an expression in terms of 'A' and 'B':

$$d_{P_1P_2} = \underbrace{\sqrt{(\cos B - \cos A)^2 + (\sin B - \sin A)^2}}_{c \text{ from cosine law}}$$

$$(\cos B - \cos A)^2 + (\sin B - \sin A)^2$$
$$= 1^2 + 1^2 - 2(1)(1)\cos(A-B)$$

$$\underbrace{\cos^2 B + \sin^2 B}_{1} \qquad \underbrace{\cos^2 A + \sin^2 A}_{1}$$

$$\cos^2 B - 2\cos B\cos A + \cos^2 A + \sin^2 B - 2\sin B\sin A + \sin^2 A$$
$$= 2 - 2\cos(A-B)$$

$$\cancel{2} - 2\cos A\cos B - 2\sin A\sin B = \cancel{2} - 2\cos(A-B)$$

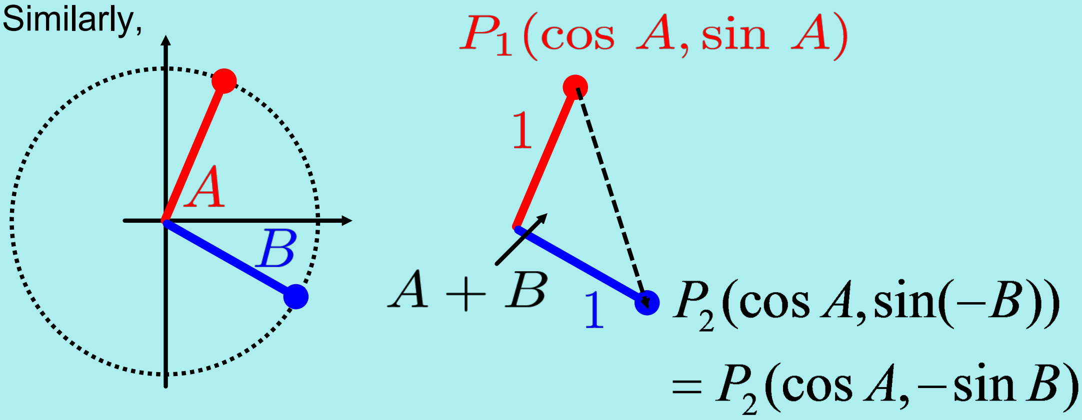
$-2 \qquad -2$

$$\cos A\cos B + \sin A\sin B = \cos(A-B)$$

$$\boxed{\cos(A-B) = \cos A\cos B + \sin A\sin B}$$

Nov 12-8:06 PM

Similarly,



$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

Nov 6-7:25 PM

Similar compound angle formulas can be obtained for sine using the complimentary angle formula:

$$\sin\theta = \cos\left(\frac{\pi}{2} - \theta\right)$$

Ex.1 $\sin(A+B) = \cos\left(\frac{\pi}{2} - (A+B)\right)$

$= \cos\left[\left(\frac{\pi}{2} - A\right) - B\right]$

$= \cos(x - y)$

$= \cos\left(\frac{\pi}{2} - A\right)\cos B + \sin\left(\frac{\pi}{2} - A\right)\sin B$

$\cos x \quad \cos y \quad \sin x \quad \sin y$

$= \sin A \cos B + \cos A \sin B$

Nov 12-8:16 PM

For the tangent function, use the quotient identity:

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

Ex.2 $\tan(A+B)$

$$= \frac{\sin(A+B)}{\cos(A+B)}$$

$$= \frac{\sin A\cos B + \cos A\sin B}{\cos A\cos B - \sin A\sin B} \times \frac{\frac{1}{\cos A\cos B}}{\frac{1}{\cos A\cos B}}$$

$$= \frac{\frac{\sin A\cancel{\cos B}}{\cos A\cancel{\cos B}} + \frac{\cancel{\cos A}\sin B}{\cancel{\cos A}\cos B}}{\frac{\cancel{\cos A\cos B}}{\cancel{\cos A\cos B}} - \frac{\sin A\sin B}{\cos A\cos B}}$$

$$= \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

Nov 12-8:27 PM

Many applications of the compound angle formulas involve angles from the special triangles.

Ex.3 Simplify and then evaluate:

$$\cos\frac{7\pi}{12}\cos\frac{5\pi}{12} + \sin\frac{7\pi}{12}\sin\frac{5\pi}{12}$$

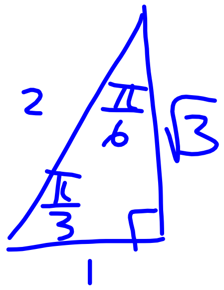
$$\cos A\cos B + \sin A\sin B = \cos(A-B)$$

$$= \cos\left(\frac{7\pi}{12} - \frac{5\pi}{12}\right)$$

$$= \cos\left(\frac{2\pi}{12}\right)$$

$$= \cos\left(\frac{\pi}{6}\right)$$

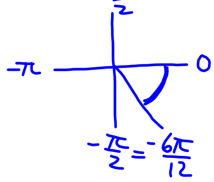
$$= \frac{\sqrt{3}}{2}$$



Nov 12-8:36 PM

Ex2 Determine an exact value for $\tan\left(\frac{-5\pi}{12}\right)$

notes:
(1) simplest to convert to RAA and apply CAST
(2) easier to see sum or difference of special angles by converting to degrees, then back to radians



$$\tan\left(-\frac{5\pi}{12}\right) = -\tan\left(\frac{5\pi}{12}\right)$$

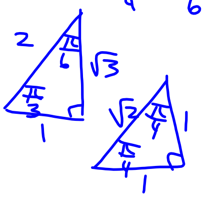
$$= -\tan\left(\frac{\pi}{4}+\frac{\pi}{6}\right)$$

$$\frac{5\pi}{12}\times\frac{180^\circ}{\pi} = 75^\circ$$

$$75^\circ = 45^\circ + 30^\circ = \frac{\pi}{4}+\frac{\pi}{6}$$

$$= -\left[\frac{\tan\frac{\pi}{4}+\tan\frac{\pi}{6}}{1-\tan\frac{\pi}{4}\tan\frac{\pi}{6}}\right]$$

$$= -\left[\frac{1+\frac{1}{\sqrt{3}}}{1-(1)\left(\frac{1}{\sqrt{3}}\right)}\right]\times\frac{\sqrt{3}}{\sqrt{3}}$$



$$= -\left[\frac{\sqrt{3}+1}{\sqrt{3}-1}\right] \quad c$$

$$= \frac{1+\sqrt{3}}{1-\sqrt{3}}$$

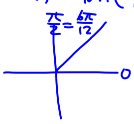
Nov 12-8:40 PM

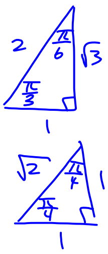
Assigned Work:

p.400 # 1-4, 5acf, 6cde, 8, 9ade, 10, 13
4e
4f f

$75^\circ = 30^\circ + 45^\circ$

4(c) $\tan\left(\frac{5\pi}{12}\right) = \tan\left(\frac{\pi}{6}+\frac{\pi}{4}\right)$





$$= \frac{\tan\frac{\pi}{6}+\tan\frac{\pi}{4}}{1-\tan\frac{\pi}{6}\tan\frac{\pi}{4}}$$

$$= \frac{\frac{1}{\sqrt{3}}+1}{1-\frac{1}{\sqrt{3}}(1)}$$

$$= \frac{\frac{1+\sqrt{3}}{\sqrt{3}}}{\frac{\sqrt{3}-1}{\sqrt{3}}}$$

$$= \frac{1+\sqrt{3}}{\cancel{\sqrt{3}}}\times\frac{\cancel{\sqrt{3}}}{\sqrt{3}-1}$$

$$= \frac{1+\sqrt{3}}{\sqrt{3}-1}\times\frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$= \frac{\sqrt{3}+1+3+\sqrt{3}}{3-1}$$

$$= \frac{4+2\sqrt{3}}{2}$$

$$= 2+\sqrt{3}$$

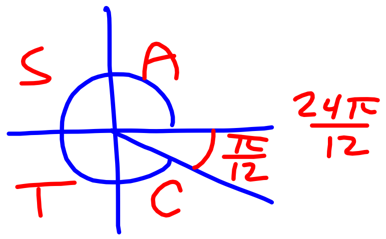
Nov 6-9:56 PM

4(f) $\tan\left(\frac{23\pi}{12}\right) = -\tan\left(\frac{\pi}{12}\right)$

$15^\circ = 45^\circ - 30^\circ$

$= -\tan\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$

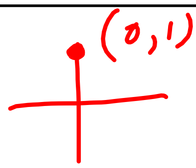
$= 60^\circ - 45^\circ$



$$= -\left[\frac{\tan\frac{\pi}{4} - \tan\frac{\pi}{6}}{1 + \tan\frac{\pi}{4}\tan\frac{\pi}{6}}\right]$$

Nov 14-9:19 AM

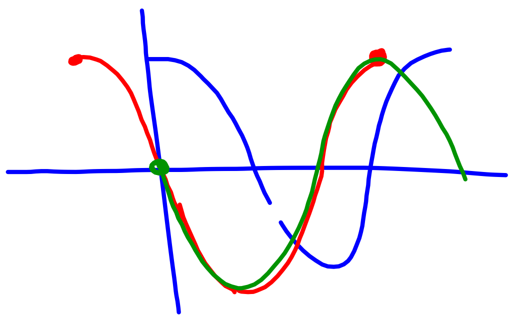
6(c) $\cos\left(x + \frac{\pi}{2}\right)$



$= \cos x \cos\frac{\pi}{2} - \sin x \sin\frac{\pi}{2}$

0 1

$= -\sin x$



Nov 14-9:22 AM

10. $\sin\alpha = \frac{7}{25}$ $\cos\beta = \frac{5}{13}$

$\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$

$y = 7$
$r = 25$
$x = \pm\sqrt{25^2 - 7^2}$
$x > 0,\ Q1$
$x = 24$

$x = 5$
$r = 13$
$y = \pm\sqrt{13^2 - 5^2}$
$y > 0,\ Q1$
$y = 12$

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$= \left(\frac{7}{\cancel{25}_{5}}\right)\left(\frac{\cancel{5}^{1}}{13}\right) + \left(\frac{24}{25}\right)\left(\frac{12}{13}\right)$$
$$= \frac{7}{65} + \frac{288}{325}$$
$$= \frac{35}{325} + \frac{288}{325}$$
$$= \frac{323}{325}$$

Nov 14-9:25 AM